

When an electric field acts upon a molecule, it will perturb its electric charge distribution.

In the simplest case, where we have just a pair of opposite charges, the electric field will displace the positive and negative charges in opposite directions. This leads to the induction of an electric dipole moment, $\hat{\mu}$

$$\text{defined as } \hat{\mu} = q \cdot \vec{r}$$

where q is the charge, and $|\vec{r}|$ is the distance between the two charges. The direction of $\hat{\mu}$ is from the negative to the positive charge. For a collection of charges we



have

$$\hat{\mu} = \sum_i q_i \cdot \vec{r}_i$$

Another way of describing the induced 2
dipole moment on a molecule when it interacts
with the light electric field \vec{E} is

$$\vec{\mu}_{\text{ind}} = \hat{\alpha} \cdot \vec{E} \quad (\text{classical description})$$

$\hat{\alpha}$ is the polarizability of the molecule and describes how easily the electrons will start oscillating (because electronic transitions occur very fast, we usually assume static nuclei).

If initially a molecule is in state ψ_1 , described by the function Ψ_1 , then the induced charge oscillation and corresponding dipole moment result in a finite probability that the molecule may "jump" to a different state, ψ_2 . This probability becomes significant only when the energy of the em wave photons equal the energy separation between the two states.

If F photon flux leaving M_1 , traveling towards M_2 , then F' after one round trip: (10)

$$F' = F e^{\sigma_{21} [N_2 - \frac{g_2 N_1}{g_1}] l} \times (1 - L_i) \times R_2 e^{\sigma_{21} [N_2 - \frac{g_2 N_1}{g_1}] l} \times (1 - L_i) \times R_1$$

$$F' = F \times R_1 \times R_2 \times (1 - L_i)^2 \times e^{2\sigma_{21} [N_2 - \frac{g_2 N_1}{g_1}] l}$$

At threshold $F' = F$ and

$$R_1 \times R_2 \times (1 - L_i)^2 \times e^{2\sigma_{21} [N_2 - \frac{g_2 N_1}{g_1}] l} = 1 \Rightarrow$$

$$e^{2\sigma_{21} N_c l} = \frac{1}{R_1 R_2 (1 - L_i)^2} \Rightarrow$$

$$2\sigma_{21} N_c l = -\ln [R_1 R_2 (1 - L_i)^2] \Rightarrow$$

$$\text{critical inversion} = N_c = -\frac{\ln R_1 R_2 + 2 \ln (1 - L_i)}{2\sigma l} \quad \left(\text{set } \sigma = \sigma_{21} \right)$$

$\gamma_1 = -\ln R_1$ $\gamma_2 = -\ln R_2$ $\gamma_i = -\ln (1 - L_i)$
 logarithmic loss of cavity mirror log internal loss of cavity

$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}$ = single pass loss of the cavity.

$$(13) \quad \boxed{N_c = \frac{\gamma}{\sigma l}}$$

once we reach N_c , oscillation (amplification) can be initiated from spontaneous emission.

Pumping schemes

How do you produce a population inversion?

We've seen that at thermal equilibrium

$$\frac{N_1}{g_1} > \frac{N_2}{g_2} \quad (\text{same as (11)})$$

When we shine light onto such a system, we would then expect that we would get more $1 \rightarrow 2$ transitions than $2 \rightarrow 1$, so maybe we could end up with population inversion. However, when we reach

$$g_1 N_2 = g_2 N_1$$

absorption and stimulated emission compensate each other, and according to eq. (9) the material becomes transparent. This is referred to

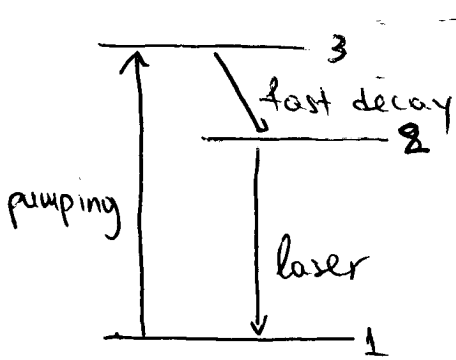
as two-level saturation

That is why we usually speak of a three-level or four-level laser.

How does that work?

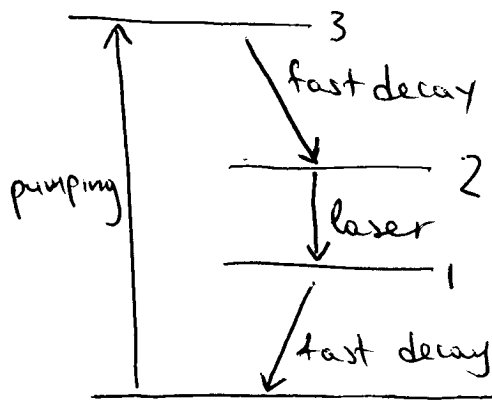
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3-level laser



Atoms are raised from level 1 (ground) to level 3. If the material is such that atoms raised to level 3, rapidly decay to level 2 (say by nonradiative decay), then we can obtain population inversion between levels 2 and 1.

4-level laser



Atoms are raised from the ground level (level 0) to level 3. Atom then decays rapidly to level 2 and we get population inversion between levels 2 and 1.

Once oscillation starts, atoms are transferred to level 1 through stimulated emission and then decay rapidly, non-radiatively to level 0.

Why 4-level laser if 3-level works?

Population inversion is much easier to achieve in 4-level system. Why?

Let's assume that at the beginning all atoms 13 are in level 1 (ground level) for 3-level system. We begin raising atoms from level 1 to level 3, which then decay to level 2. If this decay is fast enough, level 3 is essentially empty.

Let's assume levels are ^{either} ~~nondegenerate~~ or have the same degeneracy. Then, according to eq. (9) absorption losses are compensated by gain when $N_2 = N_1$. From this point on, any atom that is raised contributes to population inversion.

For a four-level system, because level 1 is empty, any atom raised to level 2 immediately produces population inversion.

Process by which atoms are raised from level 1 (or level 0) to level 3 is known as pumping. Pump rate is the rate at which level 2 is populated $R_p = \left(\frac{dN_2}{dt} \right)_p$

Properties of laser beam

(14)

- ① Monochromaticity
- ② Coherence
- ③ Directionality
- ④ Brightness.

Monochromaticity

Due to:

(a). Only an em wave with frequency ν corresponding to energy difference between levels 2 and 1 can be amplified $\nu_{\bullet} = \nu_0 = \frac{E_2 - E_1}{h}$ (14)

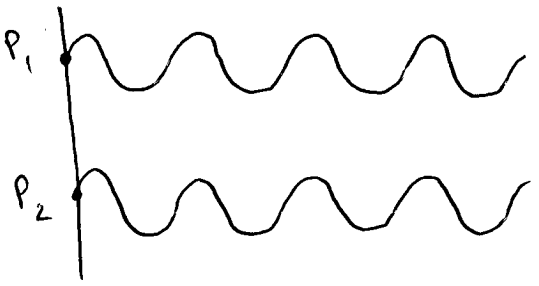
(b) Oscillation occurs at the resonant frequencies of the cavity (i.e. the length of the space between the two mirrors, L , is a multiple of the ^{half} wavelength of the emitted em wave)

$$L = n \lambda / 2 \quad (15)$$

This leads to ^{much} narrower linewidth (to order of magnitude) than the linewidth of the corresponding $2 \rightarrow 1$ spontaneous emission transition.

Coherence

Spatial coherence

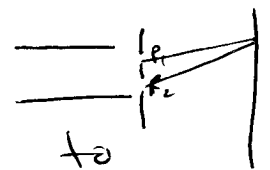


Let P_1 and P_2 be two points on the same wavefront at $t=0$.

Let $E_1(t)$ and $E_2(t)$ be the corresponding electric fields. By definition the difference in phase between these two fields at $t=0$ is zero. If the difference remains zero for $\forall t > 0$, there is perfect coherence for the two points. If such coherence occurs for any two points of the em ^{wave} front, then the wave has perfect spatial coherence. In practice, P_2 must lie within some finite area around P_1 to have good phase correlation. In this case the wave has partial spatial coherence.

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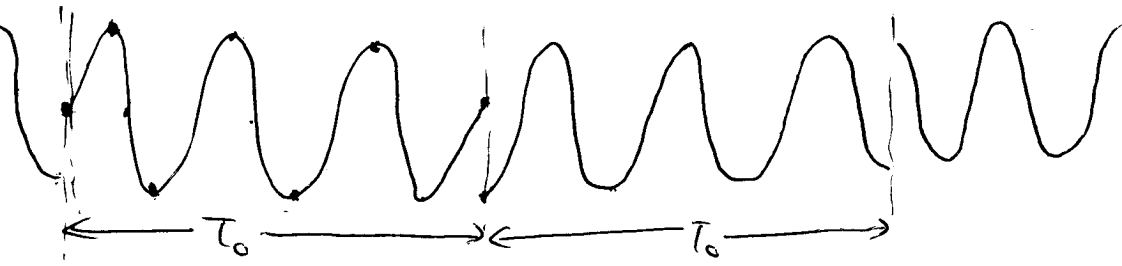
wave has partial spatial coherence.



(Young's interferometer can be used to assess spatial coherence \rightarrow how far can two slits be and produce visible fringes).

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (16)$$

Temporal coherence



Consider the electric field of the em wave at a given point P at times t and $t + \tau$.

If the phase difference for the fields at t and $t + \tau$ remains the same for any time t , then there is temporal coherence over a time τ .

If this occurs for any value of τ then the em wave has perfect temporal coherence.

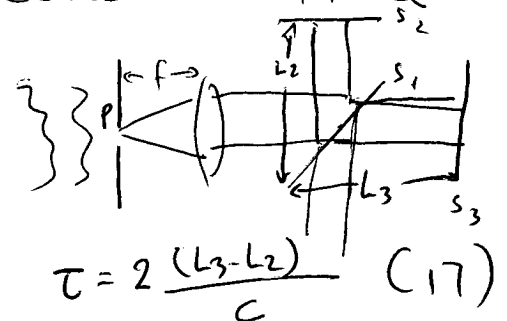
If this occurs for a time delay τ such that

$0 < \tau < \tau_0$, then the wave has partial

temporal coherence, with a coherence time

equal to τ_0 .

(Use Michelson interferometer to assess temporal coherence.
What is max τ for which interference is visible)



$$\tau = \frac{2(L_3 - L_2)}{c} \quad (17)$$

Directionality

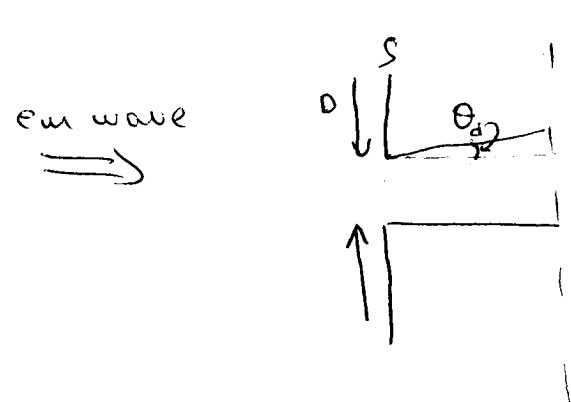
Active medium is in a resonant cavity →

for plane parallel cavity



only wave propagating orthogonal to the mirrors can be sustained in the cavity.

Even for perfect spatial coherence, a finite size beam has divergence due to diffraction.



$$\theta_d = \frac{b\lambda}{D} \quad (18)$$

λ = beam wavelength

D : beam diameter

b : coefficient on the order of λ ; depends on amplitude distribution and definition of D and θ_d .

for partial spatial coherence,

$$\theta = \frac{b\lambda}{(S_c)^{1/2}} \quad (19)$$

where S_c is coherence area, around point P' .

Brightness

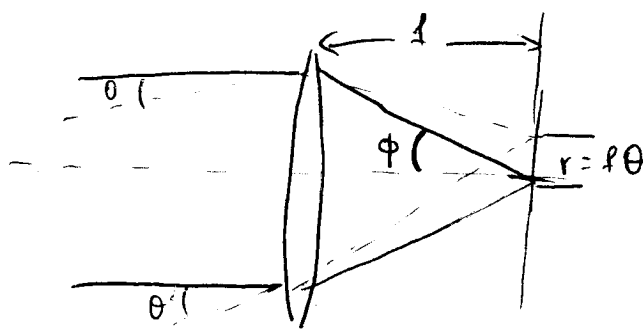
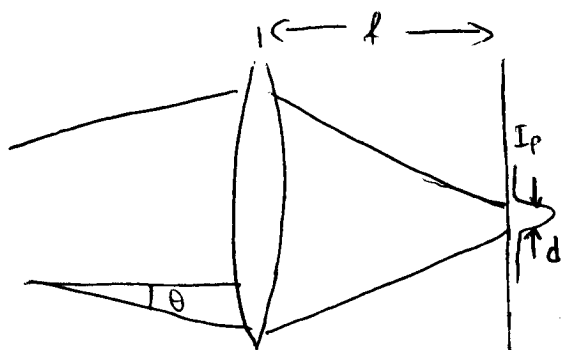
Brightness: power emitted per unit surface area per unit solid angle.

Due to highly directional properties of the laser beam.

For a laser beam

$$B = \frac{4P}{(\pi D \theta)^2} \quad (20)$$

where P is the beam power, D is the beam diameter, θ is its divergence



Brightness is probably the most important parameter of a laser beam.

For example, it is the main parameter affecting the peak intensity of the spot to which a laser beam is focused by a lens. Two parallel beams will be focused by a lens on two spots at the lens focal length separated by $r = f\theta$. Thus a beam with divergence θ will be focused into a spot with diameter $d = 2f\theta$. The peak

intensity of the beam will be

$$\bar{I}_p = \frac{P}{\pi \left(\frac{d}{2}\right)^2} = \frac{4P}{\pi d^2} = \frac{P}{\pi (f\theta)^2} \Rightarrow \text{using (20)}$$

$$\boxed{\bar{I}_p = \frac{\pi}{4} (\text{N.A.})^2 B} \quad (21)$$

N.A. = numerical aperture. = $\sin \phi =$

$$\sin(\tan^{-1}(D_L/f)) \approx D_L/f$$

D_L = lens diameter.

So, for a given lens NA, I_p depends only on B.

Short pulse duration

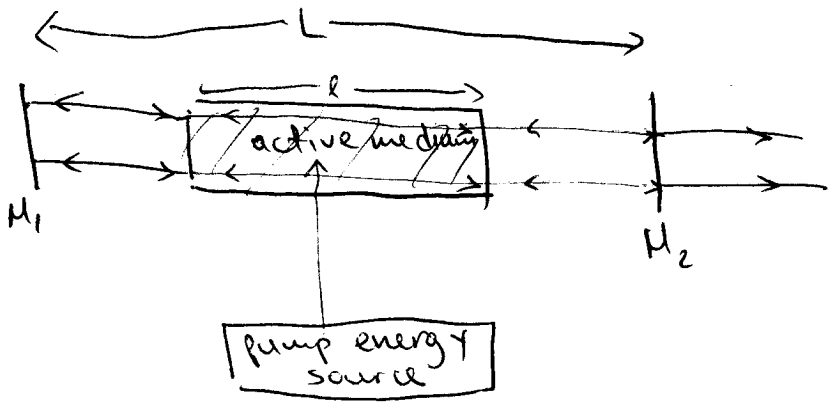
Pulses as short as a few femtoseconds can be generated by methods such as mode locking or Q switching.

For Q-switching: active Q-switches are electro-optic or acousto-optic switch which modulate the quality (transmissivity ^(Q)) of the cavity, so that you ^{continuously} build up and deplete the energy in the active medium.

passive Q-switches are typically saturable absorbers (dyes or crystals), with decreasing absorption as the intensity of the beam increases.

Pulses on the order of a few to hundreds of ns.
Mode-locking: Different longitudinal modes ($L = \frac{n \cdot d}{2}$) generated randomly in time inside the laser cavity are locked together in phase to produce a train of extremely short pulses. Modulate amplification in the cavity at a frequency $\frac{c}{2L}$, which is difference in frequency between adjacent longitudinal modes. Pulse duration achieved as low as a few fs!

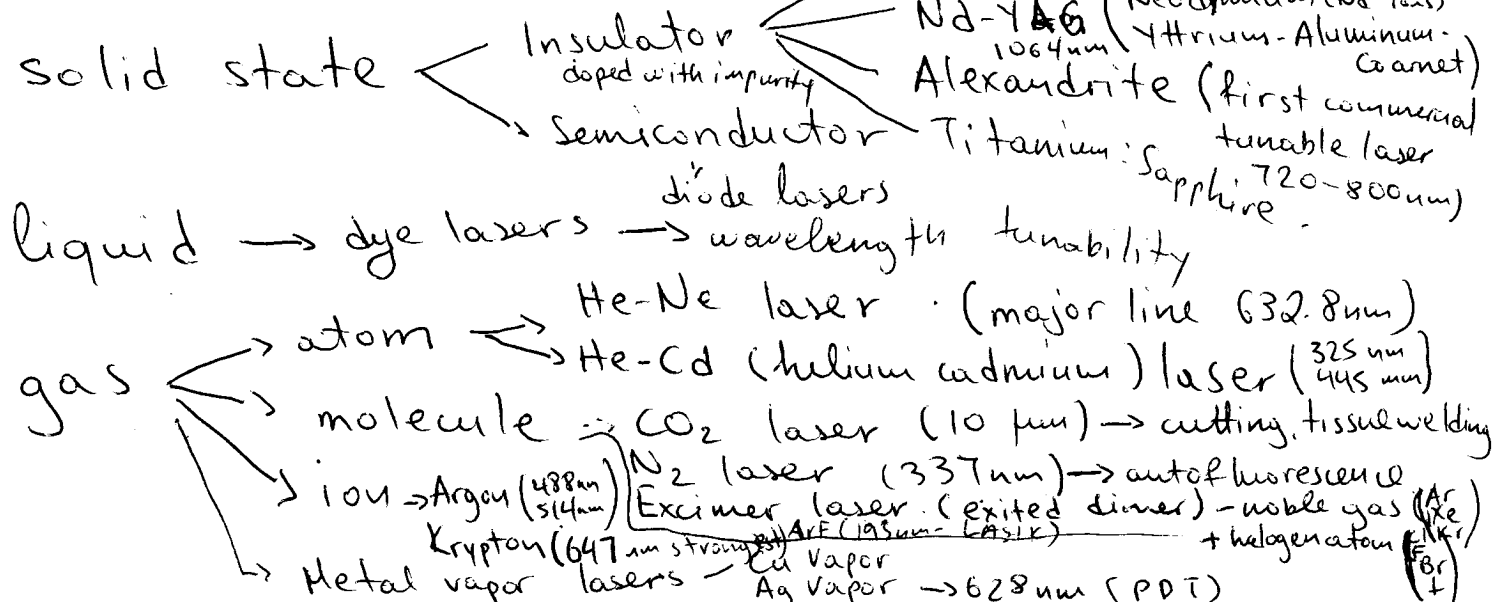
Types of lasers



Major laser components.

- ① active medium → population inversion gain.
- ② Source for pumping energy to achieve population inversion
- ③ resonant optical cavity / oscillator.

Laser types according to state of active medium:



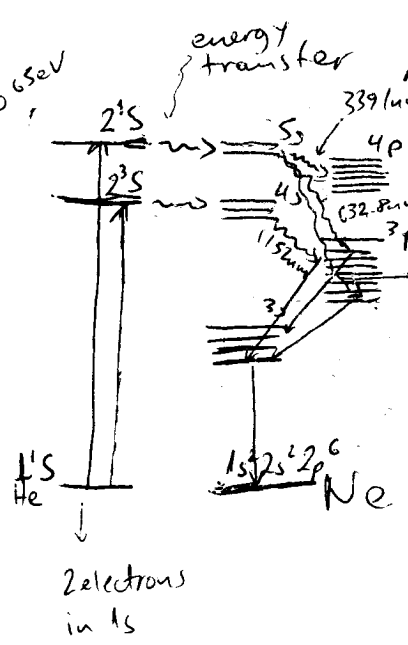
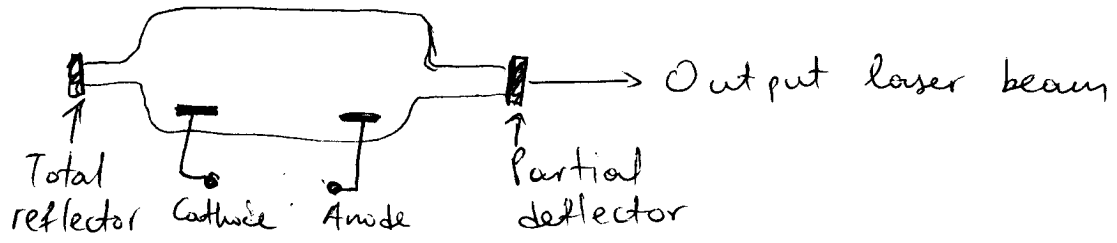
Laser type according to operation mode

- continuous wave
- pulsed (Nd-YAG) - 100 μ s \rightarrow ms pulses
- Q-switched (Nd-YAG) (Nd-YLF) - ns pulses
Yttrium Lithium Fluoride
- mode locked - Ti:sapphire \rightarrow fs pulses

Pumping process

electrical pumping:

electrical discharge through the gas



Apply a pulse of about 10KV across electrodes
 4p to start discharge \rightarrow induce an electric
 current through the gas \rightarrow excite He
 atoms by collisions with electrons in
 the discharge \rightarrow He atoms collide with
 Ne atoms, transfer their energy to them,
 and excite them to build up population
 inversion

Optical pumping
by lamp (Nd:YAG)
dye
Alexandrite

by laser dye laser
Ti:sapphire

Diode lasers

Band theory of solids

The available energy states for electrons form bands instead of discrete energy levels.

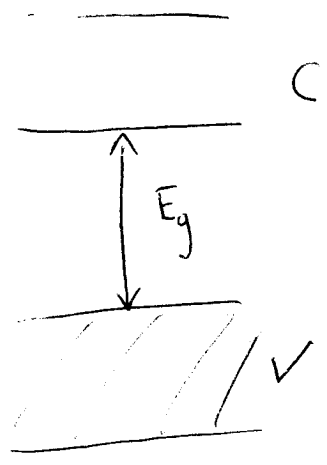
Conduction band ← upper band of allowed states. usually empty.

Valence band ← lower band of allowed states (always nearly completely filled.)

Electrons in the conduction band are free to move about the crystal

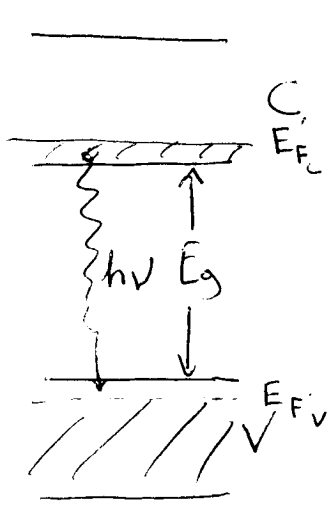
Insulators: big gap between valence and conduction band
conductors: partially overlapping bands
semiconductors: small gap, doping further decreases gap.

valence band is completely filled
 conduction band is completely empty



Suppose that some electrons are raised to C by pumping mechanism.

After short time, electrons in C drop to lowest unoccupied levels and electrons in valence band also drop to lowest unoccupied levels.



E'_{F_C} and E'_{F_V} define the energy levels for each band, below which all states are occupied by electrons. Light ^{emission} occurs when electron in conduction band falls to valence band and recombines w/ hole.

Stimulated emission can occur from this recombination, leading to laser action. For a photon

to be amplified rather than absorbed

$$E_g \leq h\nu \leq E'_{F_C} - E'_{F_V}$$

(22)

E'_{F_C} and E'_{F_V} depend on N - density of electrons raised to the conduction band, $E'_{F_C}(N) \uparrow$ with N

$E'_{F_V} \downarrow$ with N . Thus, for gain to exceed absorption losses N must exceed some critical value

$$|E'_{F_C}(N) - E'_{F_V}(N)| \geq E_g \quad (23)$$

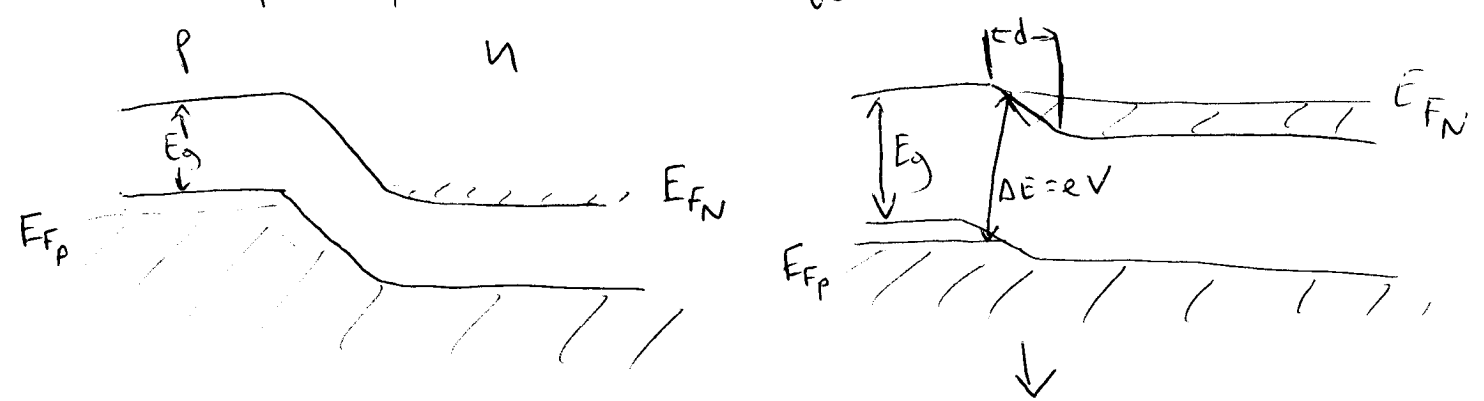
The value of injected carrier density that satisfies (23) is referred to as carrier density at transparency N_{tr}

If $N_{injected} > N_{tr} \rightarrow$ net gain in active medium. If placed in suitable cavity so that gain $>$ cavity losses \rightarrow laser.

Homojunction laser

p-type and n-type regions are of same material (GaAs). Donor and acceptor concentrations so high that Fermi level in C for

p type and in V for n type when forward voltage is applied

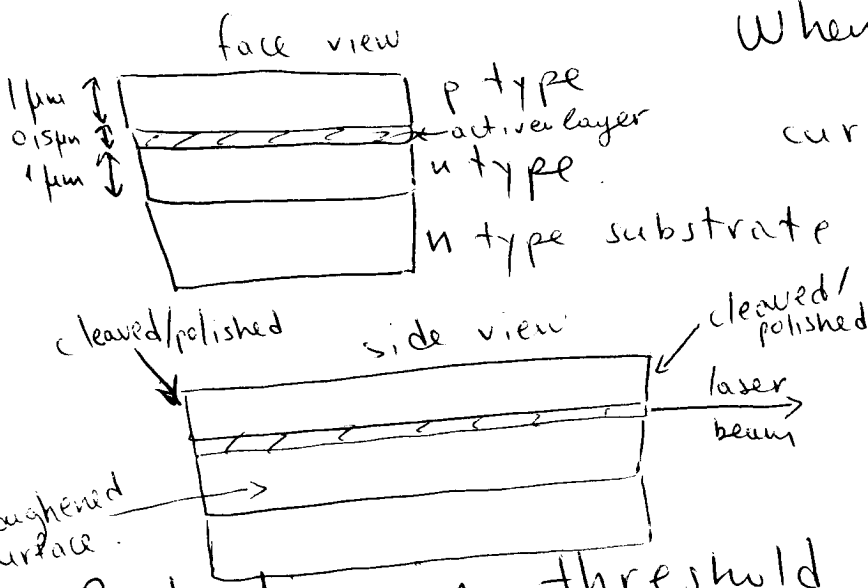


$$\bar{E}_{Fn} - \bar{E}_{Fp} > \bar{E}_g$$

Limitation: very small potential barrier encountered by electron in C band when it reaches p-side of junction \rightarrow very high threshold current density \rightarrow big penetration

Double heterostructure laser

Active medium: thin layer of GaAs or InGaAsP sandwiched between either $Al_{0.3}Ga_{0.7}As$ and InP, respectively.

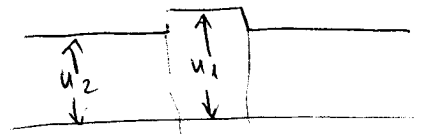


When layer thickness optimized current density threshold $\sim 10^3 A/cm^2 \sim 2$ orders of magnitude less than homojunction \rightarrow cw operation possible.

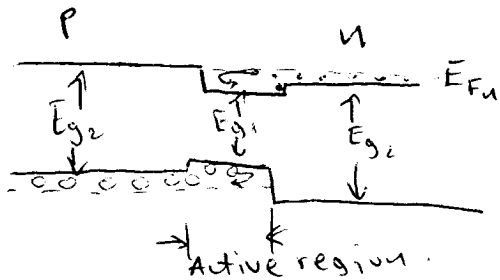
Reduction in threshold current

① photon confinement in active medium layer because of n difference

$n_{GaAs} = 3.6$ $n_{Al_{0.3}Ga_{0.7}As} = 3.4$



② The band gap in active layer E_{g1} significantly smaller than cladding layers, (E_{g2})



For GaAs: $E_{g1} = 1.5 eV$
 $E_{g2} = 1.8 eV$

\rightarrow energy barriers confining holes and electrons in active layer
carrier confinement

③ Since $E_{g2} > E_{g1}$ laser beam

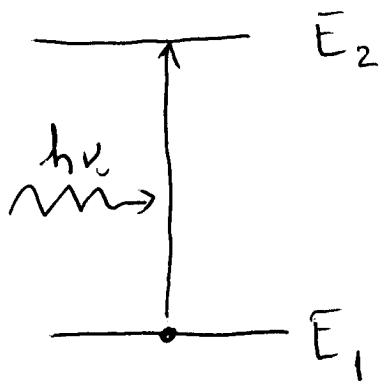
②7.

which has frequency $\nu \approx E_{g1}/h$ is less

strongly absorbed in its wings by cladding

layers \rightarrow reduced absorption

Let us consider a simple two energy level system of some atom or molecule with energy levels E_1 and E_2 . ($E_1 < E_2$). For convenience, let us assume that E_1 is the energy of the ground state, and that the atom is initially at level 1.



When an em wave interacts with such an atom, there is a finite probability that the atom will be raised to level 2 if the frequency of the em wave (ν_0) corresponds to an energy

$$E = h\nu_0 = E_2 - E_1$$

What is this probability?

Let N_i be the population of level i at time t , i.e. # of atoms/molecules/unit volume that occupy a given energy level i .

Then, we define an absorption rate W_{12} as (4)

$$\left(\frac{dN_1}{dt}\right)_a = -W_{12} N_L \quad (1)$$

where $\left(\frac{dN_1}{dt}\right)_a$ is the rate of transitions $1 \rightarrow 2$ due to absorption, and N_L is the population of

level 1.
$$W_{12} = \sigma_{12} F \quad (2)$$

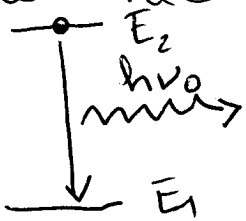
σ_{12} is the absorption cross section (units of area)
 - Also noted as B_{12} - Einstein B coefficient
 F photon flux of the wave $\left(\frac{I}{h\nu}\right)$ intensity

- sign because # of atoms leaving state.

If the atom molecule is initially at level 2. Since $E_2 > E_1$, the atom tends to decay to level 1, releasing in the process energy of $E_2 - E_1$.

When this energy is released in the form of an em wave, the process is called spontaneous or radiative emission. The frequency of

the radiative wave is



$$\nu_0 = \frac{E_2 - E_1}{h}$$

The probability that such a process will occur (5)
is defined by the following expression:

$$\left(\frac{dN_2}{dt} \right)_{sp} = -A_{21}N_2 \quad (3)$$

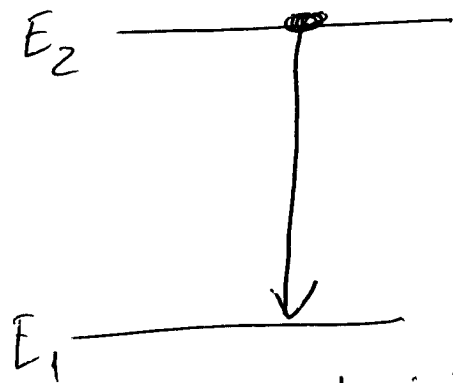
A_{21} = rate of spontaneous emission (units of time^{-1})
(Einstein A coefficient)

$\tau_{sp} = \frac{1}{A_{21}}$: spontaneous emission or radiative lifetime.

A_{21} and τ_{sp} depend only on the particular transition considered.

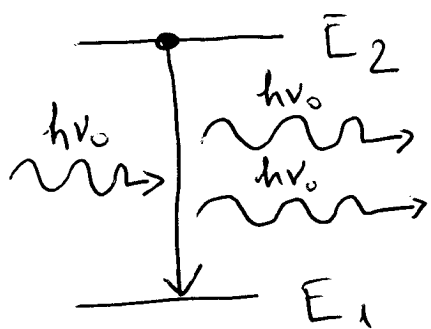
If no em wave is released during transition of the molecule from level 2 to level 1 (for example, energy is released in the form of heat) the process is called non-radiative decay. In this case

$$\left(\frac{dN_2}{dt} \right)_{nr} = -\frac{N_2}{\tau_{nr}} \quad (4)$$



τ_{nr} depends on transition AND on characteristics of surrounding medium.

If the atom/molecule is at level 2 and $\textcircled{6a}$ interacts with an em wave of frequency $\nu = \nu_0$, it results in a finite probability that the atom will undergo a transition from level 2 to level 1. In the process, an em wave will be radiated which adds to the incident wave.



We can write

$$\left(\frac{dN_2}{dt}\right)_{st} = -W_{21} N_2 \quad (5)$$

W_{21} = rate of stimulated emission
(units of time^{-1})

$$W_{21} = \sigma_{21} F \quad (6)$$

σ_{21} : stimulated emission cross-section; depends

on transition

- also noted as B_{21} - Einstein B coefficient

F = photon flux

In spontaneous emission: wave emitted by an atom doesn't have a definite phase relationship to that emitted by another atom, and can be emitted in any direction

In stimulated emission: the radiated wave adds in phase and in same direction to incident wave.

If the two levels are non-degenerate.

$W_{21} = W_{12}$ and $B_{21} = B_{12}$ $\sigma_{21} = \sigma_{12}$ if levels 1 and 2 are g_1 -fold and g_2 -fold degenerate then

$$g_2 W_{21} = g_1 W_{12} \tag{7}$$

$$g_2 \sigma_{21} = g_1 \sigma_{12} \tag{8}$$

From perturbation theory:

$$B_{21} = B_{12} = \frac{4\pi^2}{6\epsilon_0 h^2} |\vec{\mu}_{ind}|^2 \quad \epsilon_0: \text{dielectric constant}$$

$$\vec{\mu}_{ind} = \vec{\mu}_{12} = \vec{\mu}_{21} = \langle \psi_2 | \hat{\mu} | \psi_1 \rangle = \langle \psi_1 | \hat{\mu} | \psi_2 \rangle$$

$$|\vec{\mu}_{ind}|^2 = |\langle \psi_2 | \hat{\mu} | \psi_1 \rangle|^2 = \text{probability that}$$

the induced dipole will distort molecules in state 2 so that they will resemble molecules in state 1 =

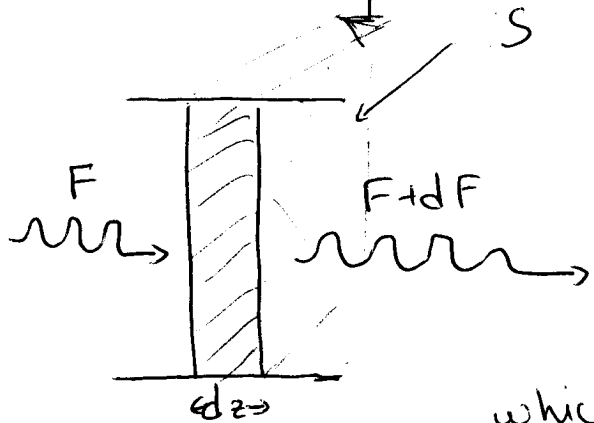
$$= |\langle \psi_2 | \hat{\mu} | \psi_1 \rangle|^2$$

$$A_{12} = \frac{8\pi h \nu^3}{c^3} B_{12}$$

Laser Idea

(7)

Consider two arbitrary levels 1 and 2 of a given material and let N_1 and N_2 be their respective populations. If a plane wave with photon flux F is traveling in the z direction in the material, then the elemental change dF along dz is due to both stimulated absorption and emission. (The photons from spontaneous emission are emitted in any direction and contribute minimally to incoming photon flux \rightarrow we ignore them)



Let S be the cross-sectional area of the beam. Then

ΔF in the shaded volume per unit time will be SdF , which should equal (using (1) and (5))

$$S dF = (W_{21} N_2 - W_{12} N_1) S dz \Rightarrow$$

$$dF = (\sigma_{21} F N_2 - \sigma_{12} F N_1) dz \Rightarrow \text{(using (8))}$$

$$dF = \sigma_{21} F \left[N_2 - \frac{g_2 N}{g_1} \right] dz \Rightarrow$$

$$\frac{dF}{dz} = \sigma_{21} F \left[N_2 - \frac{g_2 N}{g_1} \right] \quad (9)$$

(8)

Equation (9) shows that the material behaves as an amplifier (i.e. $dF/dz > 0$) if

$$N_2 > \frac{g_2 N_1}{g_1} \quad \text{and as an absorber if } N_2 < \frac{g_2 N_1}{g_1}$$

At thermal equilibrium, populations are described by Boltzmann statistics.

equilibrium populations \rightarrow

$$\frac{N_2^e}{N_1^e} = \frac{g_2}{g_1} e^{-((E_2 - E_1)/KT)} \quad (10)$$

k = Boltzmann's constant.

T = absolute temperature of material.

So in thermal equilibrium $N_2^e < \frac{g_2 N_1^e}{g_1}$ and (11)

the material is an absorber.

However, if a non-equilibrium condition is achieved

and $N_2 > \frac{g_2 N_1}{g_1}$ (12) then the material acts as an amplifier and we have population inversion.

A material in which this population inversion is produced is referred to as an active medium.

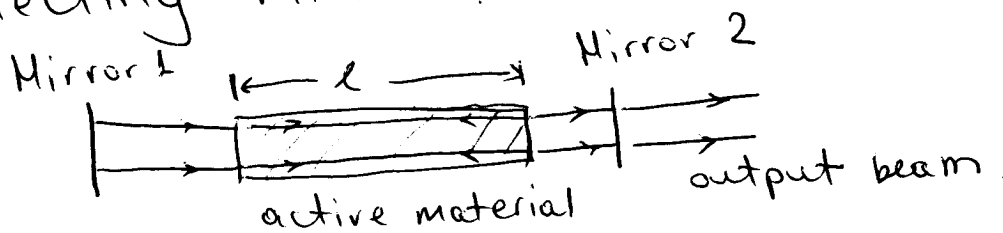
If the transition frequency $\nu_0 = \frac{E_2 - E_1}{h}$ is in the optical region the amplifier is a

laser amplifier

↓
acronym standing for light amplification by stimulated emission of radiation.

To make an oscillator from an amplifier we need positive feedback. →

place the active material between two highly reflecting mirrors.



one of the two mirrors (mirror 2) is partially transparent. The output (laser) beam is obtained from that mirror.

From (9) we have for length of active material, l
Taylor series or Maclaurin series, (2)

gain per pass in active material. $\frac{F_{out}}{F_{in}} = 1 + \sigma_{21} \left[N_2 - \frac{g_2 N_1}{g_1} \right] l \approx e^{\sigma_{21} \left[N_2 - \frac{g_2 N_1}{g_1} \right] l}$

R_1, R_2 power reflectivities of two mirrors

L_i : internal loss/pass in the laser cavity (e.g. due to output coupling).

⊛ Taylor series
Series expansion of function about a point.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin series. - Taylor series expansion
of a function about 0.

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \dots$$