Lab 4: Thin Lens Combinations

1. Read the following section about lens combinations. Two cases are presented for convex lenses.

A) Lenses separated by less than the focal length of either lens.

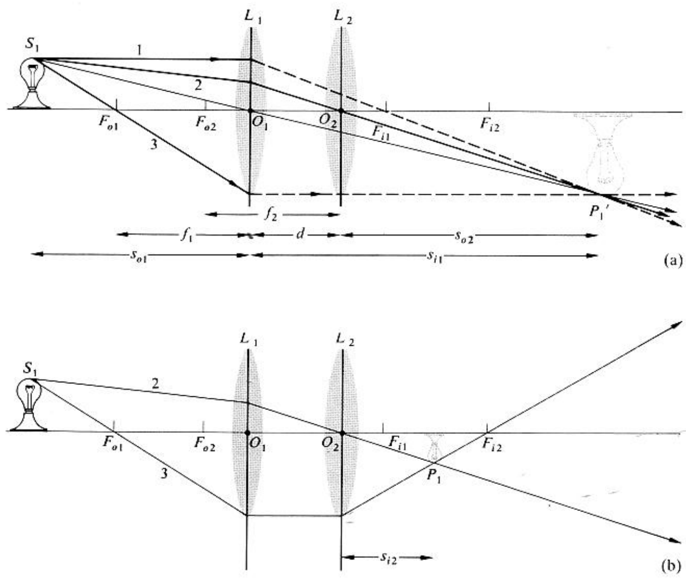

Figure 5.29 Two thin lenses separated by a distance smaller than either focal length.

Suppose we have two thin positive lenses L_1 and L_2 separated by a distance d, which is smaller than either focal length, as in Fig. 5.29. The resulting image can be located graphically as follows. If we overlook L_2 for a moment, the image formed exclusively by L_1 is constructed with rays 1 and 3. As usual, these pass through the lens object and image foci, F_{o1} and F_{i1}, respectively. The object is in a normal plane, so that two rays determine its top, and a perpendicular to the optical axis finds its bottom. Ray 2 is then constructed running backward from P_1' through O_2. Insertion of L_2 has no effect on ray 2, whereas ray 3 is refracted through the image focus F_{i2} of L_2. The intersection of rays 2 and 3 fixes the image, which in this particular case is real, minified, and inverted.

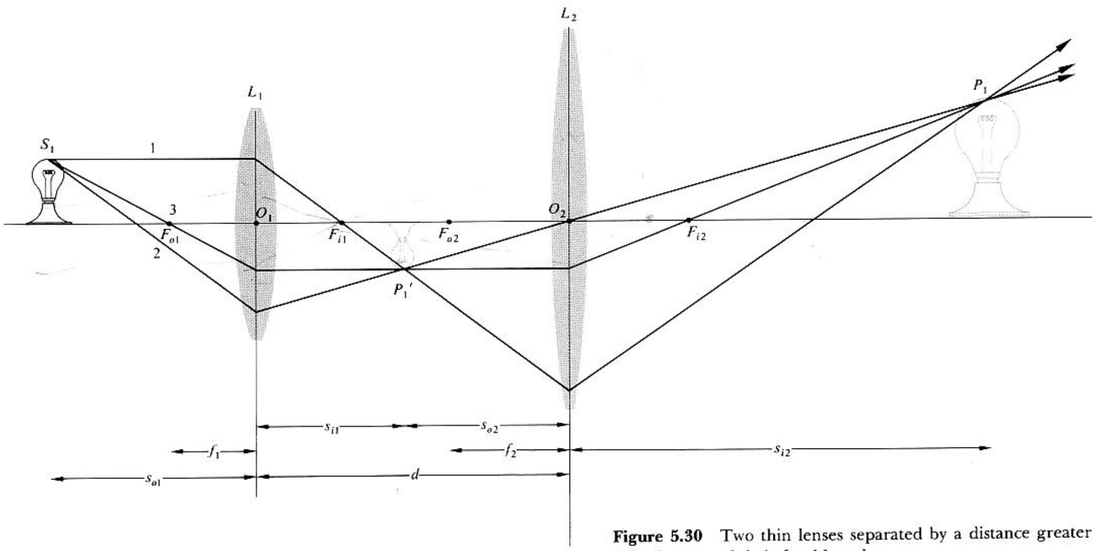

Figure 5.30 Two thin lenses separated by a distance greater than the sum of their focal lengths.

B) Lenses separated by more than the sum of the two lenses' focal lengths.

A similar pair of lenses is illustrated in Fig. 5.30, in which the separation has been increased. Once again rays 1 and 3 through F_{i1} and F_{o1} fix the position of the intermediate image generated by L_1 alone. As before, ray 2 is drawn backward from O_2 to P_1' to S_1. The intersection of rays 2 and 3, as the latter is refracted through F_{i2}, locates the final image. This time it is real and erect. Notice that if the focal length of L_2 is increased with all else constant, the size of the image increases as well.

Analytically, we have for L_1

$$\frac{1}{s_{i1}} = \frac{1}{f_1} - \frac{1}{s_{o1}} \qquad (5.29)$$

or

$$s_{i1} = \frac{s_{o1} f_1}{s_{o1} - f_1}. \tag{5.30}$$

This is positive, and the intermediate image is to the right of L_1, when $s_{o1} > f_1$ and $f_1 > 0$. For L_2

$$s_{o2} = d - s_{i1}, \tag{5.31}$$

and if $d > s_{i1}$, the object for L_2 is real (as in Fig. 5.30), whereas if $d < s_{i1}$, it is virtual ($s_{o2} < 0$, as in Fig. 5.29). In the former instance the rays approaching L_2 are diverging from P_1', whereas in the latter they are converging toward it. Furthermore,

$$\frac{1}{s_{i2}} = \frac{1}{f_2} - \frac{1}{s_{o2}}$$

or

$$s_{i2} = \frac{s_{o2} f_2}{s_{o2} - f_2}.$$

Using Eq. (5.31), we obtain

$$s_{i2} = \frac{(d - s_{i1}) f_2}{(d - s_{i1} - f_2)}. \tag{5.32}$$

In this same way we could compute the response of any number of thin lenses. It will often be convenient to have a single expression, at least when dealing with only two lenses, so substituting for s_{i1} from Eq. (5.29), we get

$$s_{i2} = \frac{f_2 d - f_2 s_{o1} f_1/(s_{o1} - f_1)}{d - f_2 - s_{o1} f_1/(s_{o1} - f_1)}. \tag{5.33}$$

Here s_{o1} and s_{i2} are the object and image distances, respectively, of the compound lens.

2. Test the case when the lenses are separated by less then either focal length. Use the 50mm and 17cm convex lenses and the lego light source. You need to do this in a very dark room to see the images. Set your 50mm lens closest to the light source.

   Where is the image located, i.e. calculate $s_{i2,}$ when d (the distance between the two lenses)= 2.5cm if s_{o1} = 20cm? What is the image orientation?

3. Test the case when the lenses are separated by more then the sum of their focal lengths. Use the 50mm and 17cm convex lenses again.

   Where is the image located when d (distance between lenses)= 25cm if s_{o1} = 20cm (calculate s_{i2})? What is the image orientation?

This is an inverted image of the lego light source filament just so that you know what you are looking for.