Lab 4: Thin Lens Combinations

1. Read the following section about lens combinations. Two cases are presented for convex lenses.

A) Lenses separated by less than the focal length of either lens.

Suppose we have two thin positive lenses $L_1$ and $L_2$ separated by a distance $d$, which is smaller than either focal length, as in Fig. 5.29. The resulting image can be located graphically as follows. If we overlook $L_2$ for a moment, the image formed exclusively by $L_1$ is constructed with rays 1 and 3. As usual, these pass through the lens object and image foci, $F_{O1}$ and $F_{I1}$, respectively. The object is in a normal plane, so that two rays determine its top, and a perpendicular to the optical axis finds its bottom. Ray 2 is then constructed running backward from $F_{I1}$ through $O_2$. Insertion of $L_2$ has no effect on ray 2, whereas ray 3 is refracted through the image focus $F_{I2}$ of $L_2$. The intersection of rays 2 and 3 fixes the image, which in this particular case is real, minified, and inverted.
B) Lenses separated by more than the sum of the two lenses’ focal lengths.

A similar pair of lenses is illustrated in Fig. 5.30, in which the separation has been increased. Once again rays 1 and 3 through $F_{11}$ and $F_{u1}$ fix the position of the intermediate image generated by $L_1$ alone. As before, ray 2 is drawn backward from $O_2$ to $P_1'$ to $S_1$. The intersection of rays 2 and 3, as the latter is refracted through $F_{12}$, locates the final image. This time it is real and erect. Notice that if the focal length of $L_2$ is increased with all else constant, the size of the image increases as well.

Analytically, we have for $L_1$

$$\frac{1}{s_{11}} = \frac{1}{f_1} = \frac{1}{s_{u1}} \quad (5.29)$$
or
\[
    s_{12} = \frac{s_{01} f_1}{s_{01} - f_1}.
\] (5.30)

This is positive, and the intermediate image is to the right of \( L_1 \), when \( s_{01} > f_1 \) and \( f_1 > 0 \). For \( L_2 \)
\[
s_{02} = d - s_{11},
\] (5.31)

and if \( d > s_{11} \), the object for \( L_2 \) is real (as in Fig. 5.30), whereas if \( d < s_{11} \), it is virtual (\( s_{02} < 0 \), as in Fig. 5.29). In the former instance the rays approaching \( L_2 \) are diverging from \( P_1 \), whereas in the latter they are converging toward it. Furthermore,
\[
    \frac{1}{s_{12}} = \frac{1}{f_2} - \frac{1}{s_{02}}
\]
or
\[
    s_{12} = \frac{s_{02} f_2}{s_{02} - f_2}.
\]

Using Eq. (5.31), we obtain
\[
    s_{12} = \frac{(d - s_{11}) f_2}{(d - s_{11} - f_2)}.
\] (5.32)

In this same way we could compute the response of any number of thin lenses. It will often be convenient to have a single expression, at least when dealing with only two lenses, so substituting for \( s_{11} \) from Eq. (5.29), we get
\[
    s_{12} = \frac{f_2 d - f_2 s_{01} f_1/(s_{01} - f_1)}{d - f_2 - s_{01} f_1/(s_{01} - f_1)}.
\] (5.33)

Here \( s_{01} \) and \( s_{02} \) are the object and image distances, respectively, of the compound lens.
2. Test the case when the lenses are separated by less than either focal length. Use the 50mm and 17cm convex lenses and the lego light source. You need to do this in a very dark room to see the images. Set your 50mm lens closest to the light source.

   Where is the image located, i.e. calculate \( s_{i2} \), when \( d \) (the distance between the two lenses) = 2.5cm if \( s_{o1} = 20\text{cm} \)? What is the image orientation?

3. Test the case when the lenses are separated by more than the sum of their focal lengths. Use the 50mm and 17cm convex lenses again.

   Where is the image located when \( d \) (distance between lenses) = 25cm if \( s_{o1} = 20\text{cm} \) (calculate \( s_{i2} \))? What is the image orientation?

This is an inverted image of the lego light source filament just so that you know what you are looking for.