STRUCTURAL ELEMENT STIFFNESS IDENTIFICATION FROM STATIC TEST DATA

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ABSTRACT: A finite element based method for static parameter identification of structures is presented for the systematic identification of plate-bending stiffness parameters for a one-third scale, reinforced-concrete pier-deck model. The plate-bending stiffnesses of the pier deck are identified at the element level by using static test data on a subset of the degrees of freedom used to define the finite element model. The finite element model of the reinforced-concrete pier deck is generated by using three-dimensional isoparametric elements to model the beams, and hourglass plate-bending elements to model the orthotropic slab.

Several parameter identification examples are performed on the pier deck using simulated static force and displacement measurements. The level of acceptable error is investigated for the convergence of the algorithm. Probabilistic parameter identification is performed to simulate an actual test setup. It is believed that continued research of this approach of static parameter identification will lead to a practical procedure for damage assessment and load-carrying capacity determination of full-scale structures.

INTRODUCTION

A large number of reinforced-concrete navy piers and bridges were built over four decades ago. In the course of the service life, these structures have sustained varying degrees of damage, including cracking, spalling, and chemical deterioration. Although some of the damage is apparent, it is difficult to evaluate the extent of the deterioration and the remaining load-carrying capacity from visual inspections. Identifying the actual plate (slab) bending stiffnesses and remaining capacity of the bridge and pier decks would classify structures in need of repair and rehabilitation.

During the summer of 1989, the Naval Civil Engineering Laboratory (NCEL) constructed a one-third scale model of a reinforced-concrete pier deck for evaluating the structural integrity of existing naval structures. The test structure is 31 ft by 18 ft (9.45 m by 5.49 m) with a 43/8-in.- (136.5-mm-) thick deck. The structure is supported by 3-ft by 18-ft (0.91-m by 5.49-m) cap beams, 12 in. (0.30 m) thick, spaced 6 ft (1.83 m) center to center, which sit upon lubricated plastic strips (Fig. 1). This paper presents a technique for evaluating the structural integrity of structures of this type. Several tasks were undertaken by NCEL, one of which was the development of this work.

A finite element software package for parameter identification of structures (PARIS) has been developed to identify the stiffness properties of orthotropic plates using static force and displacement measurements. This method of static parameter identification is capable of determining major changes in the structural properties at the element level, including element failure.

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This paper extends the parameter identification method described in Sanayei and Nelson (1986) and Sanayei and Schmolze (1987) to recover the constitutive matrix of three-dimensional orthotropic plate-bending elements from static test data. The identified constitutive matrix is used to locate the damaged areas of a reinforced-concrete pier deck.

Static Parameter Identification

With the basic assumption of a linear elastic structure, the objective of system identification is defined to be the recovery of element stiffness properties, contained in a finite element model, from static test data.

In the finite element model, the topology of the structure, the behavior of the elements, and the boundary conditions are specified at the outset. The only unknown feature of the structural model is some or all of the parameters used to describe the structural components, for example cross-sectional area, moment of inertia, or plate-bending stiffness.

In the static parameter identification algorithm, an iterative procedure is used for automatically adjusting the structural element stiffness parameters. Throughout the iterations, all the properties of the stiffness matrix, such as element connectivity, positive definiteness, symmetry, equilibrium, and bandedness, are automatically preserved.

Static parameter identification is based on the finite element method of static structural analysis, which relates forces to displacements in the usual way

$$f = Ku \quad (1)$$

where \(f\) = the vector of applied forces; \(u\) = the vector of measured displacements; and \(K\) = the global stiffness matrix.

For static parameter identification, two independent stiffness matrices are formulated: \(K^*\) is the analytical stiffness matrix derived from the finite element model; and \(K''\) is the measured stiffness matrix derived from force and displacement measurements. Both are square matrices of the same size. To derive the analytical stiffness matrix from the global stiffness matrix, Eq. 1 is partitioned into measured and unmeasured portions

$$\begin{bmatrix} f'' \\ f' \end{bmatrix} = \begin{bmatrix} K_{mm} & K_{mu} \\ K_{um} & K_{ uu} \end{bmatrix} \begin{bmatrix} u'' \\ u' \end{bmatrix} \quad (2)$$

It is assumed that there are no forces applied to the unmeasured degrees of freedom, thus

$$f'' = 0 \quad (3)$$

Eq. 2 can be simplified to

$$f'' = K'u'' \quad (4)$$

where

$$K^* = K_{mm} - K_{mu}(K_{uu})^{-1}K_{um} \quad (5)$$

It should be noted that \(K^*\) is a nonlinear function of the stiffness parameters because of the inversion of \(K_{uu}\).

Measured static forces and displacements are obtained over a limited sub-
set of the total number of the degrees of freedom used to define the finite element model. The force displacement relation obtained from the measurements may be written as

$$f'' = K''u''$$

(6)

Points on the structure where measurements are taken are chosen to match specific degrees of freedom in the finite element model. A unit force is applied to each degree of freedom selected and displacements are measured at all the chosen degrees of freedom. All the measured displacements can be arranged columnwise in a measured force displacement matrix defined as \(U''\). The unit forces can be arranged columnwise in a measured force matrix defined as \(F''\), which is equivalent to the identity matrix, \(I\). (If it is not possible to conduct the tests with unit loads, each load and corresponding displacements can be normalized for a linear elastic structure.) From these matrices, the following relationship can be established

$$F'' = K''U''$$

(7)

The measured force matrix is equivalent to the identity matrix

$$F'' = I$$

(8)

then

$$I = K''U''$$

(9)

and

$$K'' = (U'')^{-1}$$

(10)

According to the Maxwell-Betti reciprocity theorem for linear elastic structures, the matrix \(U''\) should be a symmetric matrix if the force and displacement measurements do not contain error. If the force and displacement measurements are reasonably accurate, the matrix \(K''\) may give an accurate representation of the measured substructure stiffness even though it may not be symmetric due to measurement errors. If this is the case, the effect of measurement errors can be reduced by using \(1/2[U'' + (U'')^{-1}]\) instead of \(U''\).

For the same substructure from which \(K''\) is derived, the corresponding analytical stiffness matrix \(K'(p)\) is obtained by using the finite element method as described previously. The stiffness parameters \(p\), in \(K'(p)\), are adjusted to match \(K''\) on an element-by-element basis.

To make this comparison, all the independent elements of \(K''\) and \(K'(p)\) are assembled into vectors \(k''\) and \(k'(p)\), respectively. Since both \(K''\) and \(K'(p)\) are symmetric matrices, the size of vectors \(k''\) and \(k'(p)\) is equal to the number of elements in the upper right triangle of either of these two matrices. The vectors \(k''\) and \(k'(p)\) can now be compared directly; the vector of stiffness parameters \(p\) can be adjusted on an element-by-element basis to equate the analytical response to the measured response of the structure.

In order to adjust the unknown parameters \(p\) in \(k'(p)\), a first-order Taylor series expansion is used to linearize the vector \(k'(p)\), which is a nonlinear function of the unknown parameters \(p\).

$$k'(p + \Delta p) = k'(p) + \left[\frac{\partial}{\partial p} k'(p)\right] \Delta p$$

(11)

where \(\frac{\partial}{\partial p}[k'(p)] = \) a matrix of partial derivatives of stiffnesses with respect to the parameters, which is defined as the sensitivity matrix \(S'(p)\) of the system. Eq. 11 can be written as

$$k'(p + \Delta p) \approx k'(p) + S'(p)\Delta p$$

(12)

Differentiating Eq. 5 with respect to each parameter will give the coefficients of the sensitivity matrix.

$$[S'(p)] = \left[\frac{\partial}{\partial p} K_{mn} - \left(\frac{\partial}{\partial p} K_{mu}\right) (K_{nm})^{-1} K_{nm} + K_{mn}(k_{nm})^{-1} \left(\frac{\partial}{\partial p} K_{mn}\right) - K_{mu}(K_{nm})^{-1} \left(\frac{\partial}{\partial p} K_{mn}\right)\right]$$

(13)

\(\bar{S}(p)\) is evaluated for each unknown parameter, \(i\), \(i = 1\) to the number of unknown parameters. The upper triangle of \(\bar{S}(p)\) is placed in a column vector, and the vectors are horizontally concatenated to form \(\bar{S}(p)\).

The error induced by a first-order Taylor series to represent \(k'(p + \Delta p)\) is the performance error.

$$e(\Delta p) = k'' - k'(p + \Delta p) = k'' - k'(p) - \bar{S}(p)\Delta p$$

(14)

The performance error vector can be represented by a scalar quantity defined as

$$J(\Delta p) = [e(\Delta p)']'e(\Delta p)$$

(15)

In order to minimize the scalar performance error \(J(\Delta p)\), the partial derivative of the error function with respect to each parameter is set equal to zero, \(\partial J(\Delta p)/\partial p = 0\). Thus, the following relationship can be derived.

$$\bar{S}(p)\Delta p = k'' - k(p) = \Delta k(p)$$

(16)

In general, the number of unknown parameters may be less than the number of independent measurements, in which case the sensitivity matrix in Eq. 16 will not be square. Then the method of least squares can be used to compute the unknown parameters for each iteration.

$$\Delta p = ([\bar{S}(p)']'[\bar{S}(p)])^{-1} [\bar{S}(p)']'\Delta k(p)$$

(17)

Eq. 17 can be used to set up an iterative procedure for static parameter identification.

$$p^{i+1} = p^i + ([\bar{S}(p)']'[\bar{S}(p)])^{-1} [\bar{S}(p)']'\Delta k(p)$$

(18)

In order to successfully and efficiently use the static parameter identification algorithm, several guidelines that are useful in making a physical interpretation of the mathematical phenomena observed during the execution of the static parameter identification algorithm are briefly noted here. For more details consult Sanaye (1989).

To deduce a unique set of parameters from a given set of measurements, the number of independent measurements must be greater than or equal to the number of unknown parameters. If this condition is not satisfied, there will be an infinite combination of parameters that satisfy the measurements.

The existence of a solution to Eq. 16 is determined by using elementary
row operations to reduce the augmented matrix \([S(p), \Delta k(p)]\) to row echelon form. If the rank of the sensitivity matrix is less than the number of unknown parameters, the system of equations is singular and cannot be solved uniquely. Also, in each nonzero row of the row echelon form of \(S(p)\), the existence of more than one nonzero number represents a linear dependency between parameters corresponding to these columns. The number of linear dependencies in the sensitivity matrix reduces the number of linearly independent measurements, thus reducing the number of identifiable parameters.

The linear dependencies develop for several reasons. First, each finite element is capable of transferring only limited amount of information. Second, the reduced analytical stiffness matrix may be banded because of the topology of the corresponding substructure. In this case, the analytical stiffness vector, \(k'(p)\), contains at least one inherent zero. This causes a row of zeros in the sensitivity matrix, which reduces its rank by one.

The sensitivity matrix may be ill-conditioned (nearly singular), thus solving the system of equations with an ill-conditioned sensitivity matrix results in numerical problems, which leads to an erroneous \(\Delta p\) or divergence. The ill-conditioning of the sensitivity matrix may be determined by computing the eigenvalues of the sensitivity matrix. If some of the eigenvalues are very close to zero, or if the condition number (the condition number is the ratio of the largest and smallest eigenvalues) of the sensitivity matrix is very high, the sensitivity matrix is ill-conditioned. In this case, either the number of unknown parameters should be reduced, or the number of measurements should be increased. Changing the subset of degrees of freedom measured for the given set of unknown parameters may also remove the ill-conditioning.

**Finite Element Model**

To use the parameter-identification program, a finite element model must be developed that accurately represents the behavior of the test structure. At the same time, the finite element model must be as small as possible to make it cost effective when employed in the parameter-identification program.

The pier-deck model consists of a double-reinforced orthogonal slab supported by cap beams. Parameter identification is limited to the instrumented test span 4 (Fig. 1). Earlier finite element analysis (Sanaye and Scampoli 1989) illustrates that fixing the base of beam 3 and eliminating spans 1 and 2 induces only a small error in the displacements of the instrumented test span. Thus, to save modeling costs a three-span model is investigated.

The cap beams of the bridge-deck model are very stiff when compared to the slab. Thus, it was concluded to model the beams with a three-dimensional isoparametric element (Bathe 1982). The three-dimensional isoparametric element uses trilinear interpolation functions, a 2 by 2 by 2 lattice of integration, and the material properties of 4,000 psi (27,580 kPa) concrete.

To model the flexible orthotropic slab, a plate-bending element is used (Nelson 1989). This is an eight-node element with three translational degrees of freedom per node. The three-dimensional plate-bending element is based on the decomposition of its behavior into uniform strain, hourglass (zero energy modes), and rigid body modes using underintegration techniques (one-point Gaussian quadrature). The final element stiffness matrix contains uniform strain, rigid body displacement, and uniform curvature capabilities required for thin plate-bending analysis.

The grid of the finite element mesh is selected by considering the instrumentation points provided by NCEL. Since, the location of the instrumentation points is fixed, the finite element mesh is selected so that nodal points occur at the instrumentation points. Sketches of the selected mesh are included in Figs. 2 and 3.

In order to model the orthotropic slab, the bridge deck is separated into two areas of reinforcement: zones A and B (Fig. 4). The equivalent moments of inertia (Wang and Salomon 1985) are calculated for the two zones by considering the cracked section of the double-reinforced slab in orthogonal directions. The results of the calculations are presented in Table 1.

Once the values of the equivalent moments of inertia for a reinforced-concrete cross section are found, the plate-bending stiffnesses for an orthotropic reinforced-concrete plate can be computed from the following equations, as listed in Timoshenko and Woinowsky-Krieger (1970).
FIG. 4. Zones of Reinforcement

TABLE 1. Equivalent Moments of Inertia for Uncracked Section

<table>
<thead>
<tr>
<th>Zone</th>
<th>$I_{eq}$ (in.⁴/ft)</th>
<th>$I_{eq}'$ (in.⁴/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>168.77</td>
<td>159.88</td>
</tr>
<tr>
<td>B</td>
<td>172.44</td>
<td>158.33</td>
</tr>
</tbody>
</table>

\[
D_{11} = \frac{E_{c}}{1 - v_{c}^2} \times (I_{eq}') \quad \quad \quad \quad \quad (19a)
\]
\[
D_{22} = \frac{E_{c}}{1 - v_{c}^2} \times (I_{eq}') \quad \quad \quad \quad \quad (19b)
\]
\[
D_{12} = v_{c} \sqrt{D_{11}D_{22}} \quad \quad \quad \quad \quad (19c)
\]
\[
D_{66} = \frac{1 - v_{c}}{2} \sqrt{D_{11}D_{22}} \quad \quad \quad \quad \quad (19d)
\]

where $E_{c} = 3,605$ ksi (24,855 MPa); and $v_{c} = 0.19$ for 4,000 psi (27,580 kPa) concrete.

In order to reduce the number of active degrees of freedom in PARIS, substructuring (Zienkiewicz and Taylor 1989) is performed on the finite element model of the test structure. A superelement is generated by condensing out all the internal degrees of freedom of a substructure of the finite element model. In the parameter-identification program only the stiffness matrix for the instrumented test span is generated, and the substructure stiffnesses are added to the correct degrees of freedom of the instrumented test-span stiffness matrix.

**PARAMETER IDENTIFICATION WITHOUT MEASUREMENT ERROR**

In this section the objective is to perform parameter identification on different subsets of elements to identify different types of localized damage in the test span. The goal is to select different subsets of parameters to identify the entire test span with a series of parameter-identification runs.

There are 20 elements in the test span, each containing four bending stiffness and/or two transverse shear stiffnesses. Thus, there is a maximum of 60 stiffness parameters used to model the test span. Ten measurement stations provide 55 independent measurements, thus a maximum of 55 unknown parameters can be identified.

The initial values of the unknown parameters are derived from the entries of the constitutive matrix for plate bending for an uncracked slab, and the transverse shear stiffnesses. The true values of the unknown parameters used to simulate damage in the slab are approximately 50% of the initial values. In all cases, the simulated damage is limited to stiffness deterioration of one or two elements. In each case, the damaged elements are selected to be contained within the subset of unknown elements.

Fig. 5 illustrates the element numbering and the instrumentation points of the test span. Table 2 lists each of the parameter identification examples and the results.

The aforementioned examples can be categorized by the types of identified damage. PARIS1–PARIS9 are attempting to identify only bending stiffnesses. In these examples, there are four unknowns per element. Whereas, PARIS10–PARIS12 are attempting to identify both bending stiffnesses and transverse shear stiffnesses. In these examples, there are six unknown parameters per element.

The first example PARIS1 contains 55 unknown parameters with damage to two elements. PARIS1 contains the maximum number of unknown parameters that can be identified from the test measurements. There are no apparent linear dependencies in this example, however this case did not converge because of an ill-conditioned sensitivity matrix. The ill-conditioning causes the gradient search to diverge, as the error function is not minimized for this example.

To reduce the ill-conditioning of the sensitivity matrix, the number of unknown parameters is reduced to 48. Reducing the number of unknown
Table 2: Summary of Parameter Identification Runs

<table>
<thead>
<tr>
<th>Example</th>
<th>Elements</th>
<th>NUE</th>
<th>NUP</th>
<th>ITER</th>
<th>RES</th>
<th>CN</th>
<th>E_{min}</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARIS1</td>
<td>6, 7, 9–20</td>
<td>4'</td>
<td>55</td>
<td>10</td>
<td>D</td>
<td>1.7E11</td>
<td>4.6E–15</td>
</tr>
<tr>
<td>PARIS2</td>
<td>9–20</td>
<td>4</td>
<td>48</td>
<td>10</td>
<td>C</td>
<td>1.6E08</td>
<td>4.8E–12</td>
</tr>
<tr>
<td>PARIS3</td>
<td>1–12</td>
<td>4</td>
<td>48</td>
<td>7</td>
<td>C</td>
<td>6.4E07</td>
<td>7.7E–12</td>
</tr>
<tr>
<td>PARIS4</td>
<td>5–16</td>
<td>4</td>
<td>48</td>
<td>10</td>
<td>C</td>
<td>5.6E07</td>
<td>1.2E–11</td>
</tr>
<tr>
<td>PARIS5</td>
<td>2, 3, 6, 7, 10, 11, 14, 15, 17–20</td>
<td>4</td>
<td>48</td>
<td>10</td>
<td>C</td>
<td>8.7E06</td>
<td>6.4E–11</td>
</tr>
<tr>
<td>PARIS6</td>
<td>6, 7, 9–16</td>
<td>4</td>
<td>40</td>
<td>6</td>
<td>C</td>
<td>9.2E05</td>
<td>1.3E–11</td>
</tr>
<tr>
<td>PARIS7</td>
<td>10, 11, 13–20</td>
<td>4</td>
<td>40</td>
<td>6</td>
<td>C</td>
<td>8.1E04</td>
<td>7.2E–10</td>
</tr>
<tr>
<td>PARIS8</td>
<td>2, 3, 6, 7, 10, 11, 14, 15, 18, 19</td>
<td>4</td>
<td>40</td>
<td>7</td>
<td>C</td>
<td>5.3E06</td>
<td>1.0E–09</td>
</tr>
<tr>
<td>PARIS9</td>
<td>1–4, 6, 7, 10, 11, 14, 15, 18, 19</td>
<td>4</td>
<td>48</td>
<td>10</td>
<td>D</td>
<td>1.1E08</td>
<td>4.9E–12</td>
</tr>
<tr>
<td>PARIS10</td>
<td>17–20</td>
<td>6</td>
<td>24</td>
<td>8</td>
<td>C</td>
<td>9.6E06</td>
<td>3.6E–09</td>
</tr>
<tr>
<td>PARIS11</td>
<td>1, 4, 5, 8, 9, 12, 13, 16</td>
<td>6</td>
<td>48</td>
<td>6</td>
<td>C</td>
<td>2.4E10</td>
<td>9.1E–12</td>
</tr>
<tr>
<td>PARIS12</td>
<td>5, 8, 9, 12, 13, 16, 17, 20</td>
<td>6</td>
<td>48</td>
<td>6</td>
<td>C</td>
<td>1.6E10</td>
<td>1.3E–11</td>
</tr>
</tbody>
</table>

*Element 6 of PARIS1 only contains three unknown parameters, as 55 is not divisible by four.

The results of these 12 parameter identification runs illustrate the capability of the PARIS program to locate damaged areas of the test span successfully. Forty-eight parameters were identified for a variety of test cases. The program is capable of detecting localized bending and shear stiffness degradation. In the next section, the effect of measurement errors will be investigated.

Level of Acceptable Measurement Errors

Past experience (Sanaye 1986) has shown that an acceptable level of error is dependent on the structural model, level of indeterminacy, the set of force and displacement measurements, and the subset of parameters identified. The level of acceptable measurement error is a function of the topology of the structure. Thus, the level of acceptable measurement errors is different for various problems. In this section, the goal is to determine the level of acceptable error for the pier-deck problem.

There are three possible outcomes of the input/output error relationship. The first is the linear range; small errors in the measurements correspond to small errors in the identified parameters. The second is the nonlinear range; small errors in the measurements correspond to large errors in the identified parameters. The third is the divergence range; small errors in the measurements result in algorithmic failure. The objective is to find the level of error for the pier deck that induces linear errors in the identified parameters. For this purpose, three different error levels are investigated, 0.002%, 0.02%, and 0.2% measurement errors.

PARIS7 is selected as a test case for the error analysis. This example is selected as a fair representation of the behavior of the parameter-identification algorithm. Three Monte Carlo (Hart 1982) observations are performed for each of the specified levels of error. The reason so few Monte Carlo observations are performed is the considerable amount of computer time necessary for one parameter-identification run. Due to the low number of Monte Carlo observations performed, a poor estimate of the mean will result. If the number of Monte Carlo observations is increased, the estimate of the mean will move closer to the true value of the mean.

The results illustrate the sensitivity of the identified parameters to measurement error. The first Monte Carlo experiment (0.002% measurement error) is in the linear region of input/output error relationship. The range of errors in the identified parameters is 0.01–1.5%, which is acceptable. However, this level of measurement error is too small for practical applications.

The second Monte Carlo experiment (0.02% measurement error) is also in the linear region of the input/output relationship. The range of errors in the identified parameters is 0.4–13%, which is acceptable. This level of measurement error requires very sensitive instruments.

The third Monte Carlo experiment (0.2% measurement error) shows a nonlinear relationship between measurement error and the identified parameters. The parameter-identification algorithm fails to converge for two observations. If the measurement error is increased beyond this limit of 0.2% the algorithm will probably diverge.

Thus, it is concluded that an acceptable level of measurement error for the aforementioned example (with 40 unknown parameters) is 0.02%. A 0.02% error corresponds to a standard deviation of 0.0001 for force and displacement measurements using a normal distribution for errors (Sanaye 1986).
This level of acceptable measurement error requires very sensitive measurement devices, thus it becomes more difficult to conduct the actual test measurements. The level of acceptable measurement error is a function of the topology of the structure. Since, the location of the measurement stations is fixed, the only variable of the topology of the structure that can be changed is the subset of parameters investigated. It has been shown that by reducing the subset of unknown parameters the algorithm tends to converge faster (Sanaye 1986). Thus, by reducing the number of unknown parameters, the parameter-identification algorithm may be able to tolerate a higher level of error.

The example selected for the second set of Monte Carlo experiments is a subset of PAR16, elements 9–16 are variable (32 unknown parameters) with damage to elements 10 and 15. Three different levels of error are investigated for this example 0.2%, 0.1%, and 0.05%.

The first Monte Carlo experiment (0.2% measurement error) shows a nonlinear relationship between the measurement errors and the identified parameters. The parameter-identification algorithm fails to converge for all three observations. The second Monte Carlo experiment (0.1% measurement error) is also in the nonlinear region of the input/output relationship. The third Monte Carlo experiment (0.05% measurement error) is in the linear range of input/output error relationship. The range of errors in the identified parameters is 0.02–7.1%, which is acceptable. This level of measurement error also requires very sensitive force- and displacement-measuring devices.

Thus, it is concluded that an acceptable level of measurement error for the aforementioned example (with 32 unknown parameters) is 0.05%. A 0.05% error corresponds to a standard deviation of 0.00025 for force and displacement measurements for a normally distributed random variable (Sanaye 1986). This is a low limit, however it is recommended for algorithmic convergence that the error limit not be close to the nonlinear range of the input/output relationship.

**Probabilistic Parameter Identification**

In order to simulate an actual test setup, two important limitations are assumed to exist; the number of independent measurements is usually less than the number of unknown parameters, and there exists some level of measurement error. The structural parameters are divided into two groups. In the first group, the parameters are known with a high degree of confidence and are set equal to the initial values. In the second group the parameters are unknown. Because of the existence of errors in the test measurements, the identified structural parameters will include some level of error.

In order to solve such a problem, it is only possible to consider subsets or unknown parameters. The subset of parameters may or may not contain all the damaged elements. There are two possible outcomes of a parameter-identification run. If the subset of the parameters does not contain all the damaged elements, then the algorithm may either diverge, or converge on an incorrect set of values (negative stiffnesses or stiffnesses much larger than the initial values). If the subset of parameters contains all the damaged elements, then the parameter-identification algorithm should converge on the true values of the parameters within the acceptable error range.

Thus, to perform probabilistic parameter identification, there are three basic steps to follow. The first step is to select different subsets of parameters in the area where damage is suspected. The second step is to review the outcome of each of the parameter-identification runs. Cases that diverge or converge on unacceptable values are eliminated. The remaining cases should produce consistent results of the identified structural parameters within the acceptable error tolerance. The third step is to select a new case based on the results of the preliminary runs to verify the identified damage.

For the bridge-deck problem, six different subsets of parameters are selected to perform probabilistic parameter identification on the instrumented test span. Each subset of parameters contains 32 unknown parameters corresponding to eight unknown elements. The level of measurement error is set equal to 0.05% as recommended earlier in this paper. The results of the six probabilistic parameter-identification runs are presented in Table 3.

Based on the results of the six probabilistic parameter-identification runs, scenarios one, three, and four are eliminated. The results of scenario two, five, and six are examined for structural parameters that change significantly. Table 4, showing consistent results, is generated for scenarios two, five, and six, which all converged.

Based on the tabulated results, a seventh probabilistic parameter-identification run is generated to verify the results presented in Table 4 (see Table 5). For this final run, only the bending stiffnesses of elements 10 and 15

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**TABLE 3. Summary of Probabilistic Parameter-Identification Runs**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Unknown elements</th>
<th>ITER</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13–20</td>
<td>10</td>
<td>Diverged</td>
</tr>
<tr>
<td>2</td>
<td>9–16</td>
<td>10</td>
<td>Converged</td>
</tr>
<tr>
<td>3</td>
<td>5–12</td>
<td>10</td>
<td>Diverged</td>
</tr>
<tr>
<td>4</td>
<td>1–8</td>
<td>10</td>
<td>Diverged</td>
</tr>
<tr>
<td>5</td>
<td>6, 7, 10, 11, 14, 15, 18, 19</td>
<td>9</td>
<td>Converged</td>
</tr>
<tr>
<td>6</td>
<td>2, 3, 6, 7, 10, 11, 14, 15</td>
<td>10</td>
<td>Converged</td>
</tr>
</tbody>
</table>

**TABLE 4. Summary of Consistent Probabilistic Parameter-Identification Runs**

<table>
<thead>
<tr>
<th>EL</th>
<th>Initial value</th>
<th>Scenario No. 2</th>
<th>Scenario No. 5</th>
<th>Scenario No. 6</th>
<th>True value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>49,830.7</td>
<td>25,594.1</td>
<td>26,942.9</td>
<td>25,019.8</td>
<td>24,915.0</td>
</tr>
<tr>
<td></td>
<td>9,727.4</td>
<td>3,969.6</td>
<td>5,111.5</td>
<td>5,103.6</td>
<td>4,863.5</td>
</tr>
<tr>
<td></td>
<td>52,600.5</td>
<td>26,629.0</td>
<td>25,634.1</td>
<td>26,543.9</td>
<td>26,300.0</td>
</tr>
<tr>
<td></td>
<td>20,734.7</td>
<td>9,546.6</td>
<td>9,681.6</td>
<td>8,798.3</td>
<td>10,367.0</td>
</tr>
<tr>
<td>15</td>
<td>49,830.7</td>
<td>23,460.2</td>
<td>22,854.4</td>
<td>22,351.4</td>
<td>22,423.5</td>
</tr>
<tr>
<td></td>
<td>9,727.4</td>
<td>4,531.2</td>
<td>5,598.2</td>
<td>5,770.0</td>
<td>4,377.2</td>
</tr>
<tr>
<td></td>
<td>52,600.5</td>
<td>25,303.6</td>
<td>24,355.6</td>
<td>24,892.5</td>
<td>25,290.0</td>
</tr>
<tr>
<td></td>
<td>20,734.7</td>
<td>9,198.5</td>
<td>10,002.1</td>
<td>9,086.0</td>
<td>9,330.3</td>
</tr>
</tbody>
</table>

*True value used to simulate measured force and displacement data.

Note: Number of iterations for scenario No. 2 is 10; scenario No. 5 is nine; and scenario No. 6 is 10.
are allowed to be variable with the other parameters held constant at their initial values. This scenario converges in five iterations to values that are consistent with the results obtained in the preliminary runs (within the specified error tolerance).

In conclusion, the parameter-identification algorithm performed successfully within the specified error tolerances for the instrumented test span of the bridge deck. If actual test measurements are used then the true values would be unknown. A decision on the extent of the damage present would be made based on information from Tables similar to Tables 3 and 4. If an actual testing procedure is implemented, the procedure outlined in this section should be followed, for more details consult Sanayei (1986).

CONCLUSIONS

In this paper, a methodology was devised for the systematic identification of the plate-bending stiffnesses of a reinforced-concrete pier deck. Bending stiffnesses and transverse shear terms are successfully identified from simulated incomplete static test data. The level of acceptable error is also investigated for this structure. Through this research several conclusions are reached:

The simplified hourglass plate-bending element proves to be suitable for modeling plate-bending behavior. The element formulation is both easy to program and computationally efficient.

Parameter identification is performed successfully using the hourglass plate-bending elements with simulated data. The program PARIS is capable of detecting large changes in element-stiffness properties.

The error tolerance for the scale model is determined to be 0.05%, which corresponds to a standard deviation of 0.00025 in the force and displacement measurements. Reducing the number of unknown parameters may increase the error tolerance of the parameter-identification algorithm. The identified parameters are sensitive to measurement errors for this problem. Thus, it is important to have very accurate force and displacement measurements.

Probabilistic parameter identification is performed successfully for the bridge-deck problem. The procedure requires investigating different subsets of parameters, and examining the results for consistent solutions.

It is recommended first to perform parameter identification on the undamaged structure. This would allow analysts to adjust the stiffness properties of the finite element model to represent the physical model more accurately. Thus, when performing parameter identification on the damaged structure, the adjusted stiffness values could be used.

Continued development of this approach of static parameter identification can lead to a practical procedure for the instrumentation, testing, and identification of full-scale structures. The scope of future work for this method of static parameter identification could include: (1) The implementation of this method to a full-scale steel or reinforced-concrete structure; (2) load-capacity determination of an existing structure based on the identified parameters; (3) extension of the static parameter-identification algorithm to accept two different subsets of degrees of freedom for force and displacement measurements; and (4) investigation of alternate error functions to be used in the optimization, in order to increase the measurement-error tolerance of the algorithm.

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APPENDIX I. REFERENCES


**APPENDIX II. NOTATION**

The following symbols are used in this paper:

- $CN$ = condition number;
- $D_{ij}$ = plate bending stiffnesses;
- $E_c$ = modulus of elasticity of concrete;
- $EL$ = identified element number;
- $E_{\text{min}}$ = minimum eigenvalue;
- $e(\Delta p)$ = performance error vector;
- $f$ = applied forces vector;
- $F^m$ = measured force matrix;
- $f^m$ = measured force vector;
- $f^u$ = unmeasured force vector;
- $I$ = identity matrix;
- $I^c_{\text{eq}}$ = equivalent moment of inertia, $y$-direction;
- $I^c_{\text{eq}}$ = equivalent moment of inertia, $x$-direction;
- $\text{ITER}$ = number of iterations;
- $J(\Delta p)$ = scalar performance error;
- $K$ = global stiffness matrix;
- $K^a$ = analytical stiffness matrix;
- $K^a(p)$ = analytical stiffness matrix with respect to $p$;
- $k^a(p)$ = analytical stiffness vector;
- $k^m$ = measured stiffness vector;
- $K^m$ = measured stiffness matrix;
- $\text{NUE}$ = number of unknown parameters per element;
- $\text{NUP}$ = number of unknown parameters;
- $p$ = vector of stiffness parameters;
- $\text{RES}$ = results, C-converged, D-diverged;
- $S(p)$ = sensitivity matrix;
- $S(p)$ = sensitivity coefficient matrix;
- $U^m$ = measured displacement matrix;
- $u$ = displacement vector;
- $u^m$ = measured displacement vector;
- $u^u$ = unmeasured displacement vector;
- $\Delta p$ = change in stiffness parameters; and
- $\nu_c$ = Poisson’s ratio of concrete.