Impedance Modeling: An Efficient Modeling Method for Prediction of Building Floor Vibrations

M. Sanayei\(^1\), P. Maurya\(^2\), N. Zhao\(^3\), J.A. Moore\(^4\)

\(^1\) Professor, Dept. of Civil & Env. Engineering, Tufts University, Medford, MA, (617) 627-4116, masoud.sanayei@tufts.edu
\(^2\) Graduate Student, Tufts University, Medford, MA, pradeep.maurya@tufts.edu
\(^3\) Former Graduate Student, Tufts University, Medford, MA, ningyu.zhao@tufts.edu
\(^4\) Supervisory Consultant, Acentech Inc., Cambridge, MA, jmoore@acentech.com

ABSTRACT

Buildings in major cities may be affected by vibrations from many sources such as trains, and subways. These ground-born vibrations are received at the building foundations and propagate up throughout the structures. The prediction of building floor vibrations plays an important role in the design of buildings for human comfort, the operation of sensitive equipment, and in nano-scale manufacturing. The ability to quantify and to predict building floor vibration levels enables engineers to take sufficient preventive measures based on a site condition.

The accuracy of Finite Element Analysis (FEA) used for the prediction of building floor vibration depends on the mesh size used for modeling and the frequency range of the excitations. Although fine mesh sizes in FEA leads to greater accuracy, it becomes computationally prohibitive to achieve accurate predictions in higher frequency ranges. An impedance model provides an alternative simplified technique for the prediction of building floor vibration with great accuracy even in higher frequency ranges.

This paper successfully illustrates the robustness of impedance modeling as compared to finite element (FE) modeling, in the prediction of building floor vibrations due to measured vibrational excitation at the foundation level. Impedance modeling provides the vibration response with a high degree of accuracy across the full frequency spectrum. Such models show the axial wave propagation along the length of a column when the floors are treated as point impedance discontinuities where they are attached to a column. Compared to FE models, it provides much higher accuracy with a method that is far more computationally efficient. Impedance modeling is a powerful tool for the simplified prediction of train and subway induced vibrations in the design phase of new buildings and may help with mitigating the effects of floor vibrations in existing buildings.
INTRODUCTION

Buildings that house sensitive equipment, laboratories, precision manufacturing equipment, or important facilities are susceptible to vibrations induced by subways or trains. High frequency vibrations propagating up the building columns also induce structural-born sounds. In residential setups, these undesirable floor vibrations and structural-born sounds can cause human discomfort as well. Predictions of sound and vibration levels play a vital role in the design of buildings. It enables design engineers to predict the floor vibration level to which building will be subjected, in design phase itself and hence adoption of appropriate vibration mitigation techniques as per the site condition is facilitated.

Predictions are ordinarily made by design engineers using design codes, empirical models, finite element models, and impedance models. The empirical models are usually based on data sets collected from other structures at which full scale testing was performed that are also similar to the design problem at hand. The accuracy of finite element models is very much dependent on the element mesh sizes incorporated in the model. Impedance models are simplified models of building floors and columns using wave propagation techniques.

In this paper, the predicted response of the FE model and the impedance model subjected to vibrations at the foundations level are compared with laboratory measurements of the scale model building for verification. Model response accuracy and computational time are compared for these two models.

Background. Degrande and Lombaert (2001) used Krylov’s analytical prediction model to demonstrate the free field response of a ground surface due to vibration using the dynamic reciprocity theorem applied to moving loads. They validated the model by the in-situ measurements induced by a passing of a high speed train between Brussels and Paris. Sheng et al. (2006) used wave-number based finite element and boundary element methods to predict the ground vibrations from trains.

With et al. (2006) developed an empirical model to predict train-induced ground vibration in a preliminary phase. With and Bodare (2007) studied transfer functions to predict vibrations inside a building due to train-induced ground vibrations. They proposed that if the transfer functions between ground vibrations and a building response are known, the vibrations in a similar building due to a known ground motion may be predicted. Correia dos Santos et al. (2010) used the coupling of Green’s function in finite elements to validate a numerical methodology formulated in the time domain for the prediction of vehicle induced vibrations.

Sanayei et al. (2011) developed and validated an impedance based modeling technique to predict floor vibrations in buildings. Experiments using a shaker installed at the base of a building model had been carried out to create white noise in the range of 100 Hz to 5 kHz.

Building Model Configuration. Zhao et al. (2010) constructed a two-bay by two-bay four story scale building as shown in Figure 1. Type 6105-T5 aluminum alloy was used for all side beams and columns with medium-density fiberboard (MDF) used for
floor slabs. The properties of the materials were taken from the values provided by
the manufacturers and verified by laboratory component testing.

The height from the foundation to the first floor is 17 in (431.8 mm) while the
upper floors columns heights are 15 in (381 mm). Aluminum column material
properties have a Young’s modulus of 70.3 GPa (10,200 ksi), density of 2,691 kg/m³
(172.8 lbm/ft³), structural loss factor of 0.002, and Poisson ratio of 0.33.

MDF slabs in the scale model building were 4 ft x 6 ft (1.219 m x 1.829 m)
with a thickness of 0.75 in (19.1 mm) at all floor levels. The MDF floor slabs have a
Young’s modulus of 3.15 GPa (450 ksi), density of 700 kg/m³ (43.2 lbm/ft³),
structural loss factor of 0.02, and Poisson ratio of 0.18.

The center column-slab connections consisted of 8 L-shape 80/20 model 25-
4108 brackets fixed above and below floor slabs to provide stiff translational and
rotational coupling between the floors and columns (80/20 Product Manual, 2006).
The impedances of bracket masses are significant at higher frequencies which affect
the wave propagation through the columns.

The aluminum beams were fixed to the edges of floor slabs using 10-32
machine screws and connected to edge columns using 80/20 model 25-4136 brackets.
Side columns stand on composite shims sitting on 2” (51 mm) of sand and are
secured in buckets with sand placed around them. The foundation is intended to
absorb vibration energy at the base of the columns without reflecting a large amount
back; this is similar to the behavior of foundations in actual buildings.

**Building Model Instrumentation.** A Brüel & Kjaer Permanent Magnetic Vibration
Exciter Type 4808 (shaker) was connected to the base of the center column with a
Force Gauge model 8230 for force measurements. The shaker sits on four neoprene
base isolators used to ensure that only the center column is excited and to avoid
exciting the laboratory floor. Side columns are assumed to serve as boundary
conditions and do not transmit vibration from the laboratory floor to the upper floors.
in the mathematical model, and so isolation of the shaker is necessary. The shaker is driven by a Power Amplifier type 2719 rated at 180 Volt-ampere.

Acceleration measurements were made using five PCB® accelerometers, model number 352C65, in the vertical direction on the center column at each floor, as well as at the base of the column. Accelerometers were attached to 80/20 aluminum angles model 25-4108. PCB accelerometers were selected due to their flat frequency response to high frequencies and low weight.

Vibration experiments were performed using white noise in the range of 100 Hz to 5 kHz. This frequency range was used to compensate for the scale of the experimental building model compared to typical full-scale four story buildings. Since the scaling factor was 1/10, the frequency range translated to 10 to 500 Hz for a full scale building. Frequency of 500 Hz is well above the actual subway and train-induced frequencies that are verified with measurements.

FINITE ELEMENT MODELING

The scale model building has been modeled with finite elements. Three models with different mesh sizes have been constructed using the finite element program SAP2000® (2011). Floor slabs are modeled with plate bending elements and columns with axial elements. The coarse, medium and fine mesh size models have been named as Model A, Model B, and Model C respectively and are shown in Figure 2. Mesh size details are presented in Table 1.

![Figure 2. Finite Element Models A, B and C (SAP2000®)](image)

<table>
<thead>
<tr>
<th>Component Size</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Elements (in)</td>
<td>1.0</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Beam Elements (in)</td>
<td>12.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Slab Elements (in)</td>
<td>12.0 x 12.0</td>
<td>3.0 x 3.0</td>
<td>1.5 x 1.5</td>
</tr>
</tbody>
</table>

The masses of the accelerometers and bracket connections have also been added to the finite element model on all floors at the center column. The measured force from the physical building model experiment is used as input to the FE model at the base of the center column in each model. The three mesh sizes were considered so
that the effect of the mesh size on the response could be studied. The responses from FE models for each floor have been compared to the measured floor vibration responses from the physical experiment in order to observe the accuracy and conclude which model can accurately capture the response of the building in the frequency range of interest. The best FE model is to be used for comparison with the impedance model and the measured responses.

Figure 3 shows the measured force of the shaker at the base of the center column of the scale model building in the range of 100 Hz to 5 kHz. It is used as an input to the FE model at base of the centre column. The same measured force is to be used as input to the impedance model.

![Figure 3. Measured Force from Scale Model Building](image)

The building floors are absorbing energy from the column and dissipating the energy due to the damping within the MDF floor. Energy is dissipated at the bracketed column-slab connection and energy is transmitted out of column to floor with subsequent dissipation. The system damping of the built-up scale model building was not experimentally measured. Damping properties are required for computing the response of the building subjected to the measured force spectrum in Figure 3.

Proportional damping was used for modeling the energy loss of the built-up system for steady state analysis in SAP2000®. It is assumed that damping is only proportional to the stiffness matrix, \( [C] = \alpha[K] \), where \([C], [K]\) and \(\alpha\) represent internal material or hysteresis damping matrix, stiffness matrix, and stiffness coefficient, respectively. As a result stiffness coefficient \(\alpha\) will be equal to the structural loss factor \(\eta\) where it is two times modal damping ratio, \(\zeta\). In order to incorporate the system damping ratio into the FE model, a parametric study with the fine mesh Model C is also done. Preliminary calculation of responses of the FE models of the scale model building showed some resonance behavior in the lower frequency range since initially zero damping was used. The parametric study showed that the structural loss factor \(\eta\) of 0.10 controls the effect of resonance on response to provide closer match to the measured response. Thus the value of 0.10 is adopted as system structural loss factor for studying the response of FE model.

Models A, B and C were run using a high end desktop personal computer with Intel i7-2600 CPU, 3.4 GHz, 16 GB RAM with 64 bit OS. The computational time required for each model is presented in Table 2. It is observed that with the increase in the number of elements (decrease in mesh size) of FE model, the computation time increased drastically. For real buildings when the structure is comparatively large and complex, the computation time required for FE analyses may become prohibitive.
Figure 4 Comparison of Models A, B and C Responses with Measured Response
Table 2. Computation Time of Finite Element Models

<table>
<thead>
<tr>
<th>Computation Time</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
</tr>
</thead>
<tbody>
<tr>
<td>hr:min:sec</td>
<td>00:12:53</td>
<td>01:12:39</td>
<td>06:12:24</td>
</tr>
</tbody>
</table>

Figure 4 shows the comparison of FE Model A, Model B and Model C with the measured velocity responses of the physical building. The velocity response was presented in decibels (dB). From the responses of FE models A, B and C, it can be observed that as the mesh sizes change from coarse to medium, to fine, the higher frequency responses are captured much better. At high frequencies the wave lengths are shorter; as a result the higher resonance frequencies cannot be captured by a coarser spacing in the FE mesh. It can thus be concluded that Model C with the finer mesh sizes captures the building response better than coarser models A and B. It is clear that Model C velocity response predictions converge on the measured response even in the higher frequency range. Hence Model C will be used in other comparisons in this paper.

In the second half of this paper, a summary of impedance modeling method is presented for prediction of the model building responses using measured forces in Figure 3.

IMPEDEANCE MODEL WITH INPUT FORCE (WAVE PROPAGATION)

Impedance-based modeling is the wave propagation representation of vibration transmission. Each finite segment of a column is characterized by the impedances at the top and bottom of each segment. The impedance of each floor is added as input impedance at the interface between the column and the floor. It is assumed that the dominant mode of transmission of subway- and train-induced vibrations into the upper floors of buildings is from axial waves in the columns. Floors attached to the columns radiate energy as transverse bending waves.

The force-displacement relationship for an axial rod is represented by the elements of the dynamic stiffness matrix. Complete derivation of (1) is shown by Sanayei et al. (2011) to be,

\[
[k_{col}] = \frac{E_c A \beta}{\sin(\beta L)} \begin{bmatrix} \cos(\beta L) & -1 \\ -1 & \cos(\beta L) \end{bmatrix} = j \omega z_{col}
\]

(1)

where \(z_{col}\) is the impedance matrix of a finite column and, \(\omega\) is the circular frequency, \(L\) is the column length, \(A\) is the cross sectional area, \(E_c\) is the complex modulus of elasticity, and \(\beta\) is the wave number. The wave number and its relationship to wave speed \(c_L\) is (Cremer et al., 1988),

\[
\beta = \omega \sqrt{\frac{\rho}{E_c}} = \frac{\omega}{c_L}
\]

(2)
Due to impedance modeling, \([k_{col}]\) in (1) is not a static stiffness matrix. It is the frequency dependent dynamic stiffness matrix that represents all of the properties of a finite column (stiffness, damping, and mass).

Floor slabs are modeled as thin, uniform plates subject to a harmonic point load. For the case of an isotropic, infinite thin plate subjected to a point load, the driving point impedance using Kirchhoff plate theory is (Kirchhoff, 1850; Cremer et al., 1988),

\[
z_{\text{Slab}} = 8\sqrt{D\rho h} = 8h^2 \sqrt{\frac{E_c \rho}{12(1-\nu^2)}} = j\omega m_{\text{Slab}}
\]

(3)

In this case, \(E_c\) is defined as the complex modulus of elasticity of slab representing energy dissipation as,

\[
E_c = E(1+j\eta)
\]

(4)

where \(\eta\) denotes the structural loss factor of slab. For a plate of uniform thickness \(h\), the bending stiffness, \(D\), and poison ratio, \(\nu\) is,

\[
D = \frac{E_c h^3}{12(1-\nu^2)}
\]

(5)

For a building subjected to train-induced vibration, ground vibrations excite all of the columns in the building; however the current model only represents transmission from a single column. Contributions from individual columns are assumed to be statistically independent or incoherent, so that the total response of the floor is the sum of incoherent contributions from each column. Therefore it is sufficient to consider only one column in the mathematical model.

For system modeling, infinite or finite floors are interconnected by finite length columns. For example, a 4-story building system model, shown in Figure 5 as a five degree of freedom system (DOFS) (including the DOF at the base), is related to the dynamic column stiffness (1) and the dynamic mass of the floor (3). Dynamic behavior of the floor slabs driven by the vibrating column is modeled as a dynamic masses calculated based on the floor impedances. In matrix form, the frequency dependent system of equations is,

![Figure 5. Impedance Model of a 4-Story Building](image-url)
\[
[K_{\text{Col}}] \{U\} - \omega^2 [M_{\text{Slab}}] \{U\} = \{F\}
\]

However, \([K_{\text{Col}}]\) and \([M_{\text{Slab}}]\) do not represent the standard system stiffness and system mass matrices used in finite element analysis. The dynamic stiffness, \([K_{\text{Col}}]\), is assembled using only the column properties in (1). The dynamic mass, \([M_{\text{Slab}}]\), is assembled using only the slab properties in (3). The system dynamic stiffness of the structure is \(([K_{\text{Col}}] - \omega^2 [M_{\text{Slab}}])\) of size \(N\) by \(N\), where \(N\) is the number of kinematic degrees of freedom. The sizes of the system matrices represented in Figure 5 are \(5 \times 5\) (one for modeling of the column excitation at the base and four for the point impedances of the four floors). This leads to a highly computationally efficient method.

The steady state response of the system to a harmonic excitation at a given frequency is,

\[
\{U\} = [K_{\text{Col}} - \omega^2 M_{\text{Slab}}]^{-1}\{F\}
\]

Figure 6 shows the comparison between the responses from the fine FE mesh Model C, the impedance model, and the physical building model. It is clear that the impedance model predicts the response with great accuracy even in higher frequency ranges with far fewer elements. Both the FE Model C and the impedance model velocity responses match with the measured responses of the laboratory scale model building. It is observed that compared to the FE model C, the impedance model represents a closer match with the measured response.

The computation time required for the FE model increases many folds with the fine meshing. Table 3 shows the comparison of computational time requirement of FE models and impedance model. It is interesting to note that the impedance model used only 8 elements with 5 degrees of freedom and the computation time required was only 20 seconds using 4,000 frequency data points.

<table>
<thead>
<tr>
<th>Table 3. Computation Time of Impedance and Finite Element Models</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>No. of Elements</td>
</tr>
<tr>
<td>No. of DOFs</td>
</tr>
<tr>
<td>Computation Time(hr:min:sec)</td>
</tr>
</tbody>
</table>

It is observed that the impedance model is more accurate than the FE models A, B and C. It is expected that FE modeling of full scale buildings to be computing prohibitive. The impedance model will still use a one column model that is computationally more efficient. Impedance modeling has proven to be a superior method for prediction of floor vibrations for design and mitigation purposes.
Figure 6 Comparison of Model C & Impedance Model with Measured Response
CONCLUSIONS

In order to incorporate the appropriate floor vibration prediction and mitigation into design phase, it is important to estimate the train- or subway-induced vibration levels of full-scale buildings. The impedance model floor vibration prediction technique enables engineers to estimate the vibration level at all floor levels in buildings subjected to such vibrations. Based on the comparison of the three methods of floor vibration assessments, the authors conclude the following points:

1. The impedance model predicted the building floor vibration with higher accuracy, even in high frequency ranges, than the FE models. This is because FE models are limited by mesh size in capturing high frequency vibrations.

2. Although the FE model with a fine mesh predicted an accurate response in the high frequency range, it required a large increase in computation time.

3. The impedance model requires far less computing time compared to the FE model and results in highly accurate response predictions.

Through this research, it was demonstrated the robustness of the impedance model presented for the prediction of floor vibrations. It is highly accurate in predicting the vibration levels and is highly computationally efficient.

ACKNOWLEDGEMENTS

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REFERENCE
