Automated finite element model updating of a scale bridge model using measured static and modal test data

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A B S T R A C T

Structural Health Monitoring (SHM) using nondestructive test data has become promising for finite element (FE) model updating, model verification, structural evaluation and damage assessment. This research presents a multiresponse structural parameter estimation method for the automated FE model updating using data obtained from a set of nondestructive tests conducted on a laboratory bridge model. Both stiffness and mass parameters are updated at the element level, simultaneously. Having measurement and modeling errors is an inevitable part of data acquisition systems and finite element models. The presence of these errors can affect the accuracy of the estimated parameters. Therefore, an error sensitivity analysis using Monte Carlo simulation was used to study the input–output error behavior of each parameter based on the load cases and measurement locations of the nondestructive tests. Given the measured experimental responses, the goal was to select the unknown parameters of the FE model with high observability that leads to creating a well-conditioned system with the least sensitivity to measurement errors. A data quality study was performed to assess the accuracy and reliability of the measured data. Based on this study, a subset of the most reliable measured data was selected for the FE model updating. The selected subset of higher quality measurements and the observable unknown parameters were used for FE model updating. Three static and dynamic error functions were used for structural parameter estimation using the selected measured static strains, displacements, and slopes as well as dynamic natural frequencies and associated mode shapes. The measured data sets were used separately and also together for multiresponse FE model updating to match the predicted analytical response with the measured data. The FE model was successfully calibrated using multiresponse data. Two separate commercially available software packages were used with real-time data communications utilizing Application Program Interface (API) scripts. This approach was efficient in utilizing these software packages for automated and systematic FE model updating. The usefulness of the proposed method for automated finite element model updating at the element level is shown by being able to lead to simultaneous estimation of the stiffness and mass parameters using experimental data.

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1. Introduction

According to the American Society of Civil Engineers 2013 Infrastructure Report Card, “Over two hundred million trips are taken daily across deficient bridges in the nation’s 102 largest metropolitan regions.” One in nine, or below 11%, of the nation’s bridges are classified as structurally deficient, while an average of 607,380 bridges are currently 42 years old. Currently, 24.9% of the nation’s bridges have been defined as functionally obsolete [1]. The Federal Highway Administration (FHWA) estimates that the current cost to repair or replace the deficient bridges eligible under the Federal Highway Bridge Program is almost $76 billion [2]. Damage can accumulate during the life of the structure due to normal use or overuse and overloads. In the inspection process, many of the structure’s elements are not observable to the inspector since they are covered by non-structural elements or are not easily accessible, which can cause uncertainty in their final reports. To add more confidence in our current methods of inspection and to improve the maintenance of infrastructures, objective and quantifiable structural health monitoring is essential in this field.

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Implementing a damage identification strategy for aerospace, civil, and mechanical engineering infrastructures is referred to as Structural Health Monitoring (SHM). There are two main approaches to SHM: (a) non-model-based approach and (b) model-based approach [3]. Both approaches have been successfully used for damage detection in structural applications. The non-model-based approach relies on the signal processing of experimental data, while the model-based approach relies on mathematical descriptions of structural systems [4]. The alternatives to the non-model-based method include: modal analysis, dynamic flexibility measurements, matrix update methods and wavelet transform technique, which are used to determine changes in structural vibration to identify damage [5].

The model-based approach is usually implemented by using a computer model of the structure of interest, such as a Finite-Element Method (FEM), to identify structural parameters based on the measured test data. In the model-based method, the first step is to create the FE model based on the design calculations. In most cases, the initial model cannot accurately predict the actual parameters and responses of the structure; this may arise from the simplification in the modeling process. The model-based methods are often used for structural evaluations at the element level. The recent advancements in data acquisition systems (DAQ) and sensor technology have significantly improved our confidence in nondestructive test (NDT) data. Measured responses of the structure based on the experimental data can be used to validate the initial structural model and to update the parameters of the assumed model. The model calibration process involves selecting a small number of model parameters that have uncertainty so that their values cannot be known a priori and using various procedures to find the values for which their measurements best match the model predictions [6]. A promising method is to minimize the residual between the predicted response from the initial model and the measured response of the actual structure according to the observed data sets. Different approaches, methods, and technologies for effective practice of structural health monitoring were surveyed in [7]. In this book, model calibration was classified based on the selection of parameters to estimate, available measured data, formulation of the objective function, an optimization approaches, and the level of uncertainties. Using a model-based approach allows modeling and estimating the physical properties using commercially available software to create and maintain the structural model.

Robert-Nicoud et al. [8] performed the model calibration of the Lutrive Highway Bridge in Switzerland, a 395 m three span bridge with a maximum span of approximately 130 m, based on static measurements such as deflections, rotations, and strain. Bodeux and Golinv [9] successfully applied the Auto-Regressive Moving Average Vector (ARMAV) method to identify frequencies and mode shapes of the Steel-Quake benchmark, a two story steel frame structure. Koh et al. [10] presented a combination of genetic algorithms (GA) and local search techniques to identify structural parameters using vibration measurements. Hybrid structural identification methods were performed to estimate the structural parameters of a structure with 52 unknown parameters. Huang et al. [11] performed the system identification of the dynamic properties of a three-span box-girder concrete bridge using the Ibrahim Time-Domain (ITD) technique based on the free vibration test results of the bridge. Structural identification procedure was successfully applied on a three-span post-tensioned reinforced concrete (RC) highway overpass by Morassi and Tonon [12] based on modal analysis and FE model updating. Several other researchers have tackled different model updating techniques and their applications on various structures, such as Wang et al. [13], Mosawi et al. [14], Brownjohn et al. [15], Jaishi et al. [16] and Ancich [17].

Along with the analytical studies, a feasible way to verify a new methodology for structural health monitoring and system identification is to apply it first to a scale model structure. Therefore, many different test structures have been utilized for verification of SHM methods. In one of these studies, Catbas et al. [18] stated: “the main purpose of constructing this laboratory model was to close the gap between very simple laboratory tests and the field tests.”

To provide an abstract representation of a short to medium span highway bridges, the University of Central Florida (UCF) benchmark was designed to have the lower natural frequencies in the range of 1–50 Hz, similar to full scale short to medium span bridges. The UCF benchmark was used in the experimental validation of various methodologies by different researchers such as Scian della and Christenson [19], and García-Palencia and Santini-Bell [20]. Although scaling is not the focus of this research, more information on scaling problem with respect to geometrical dimensions and material properties is given by Harris and Sabnis [21].

### 1.1. Objective, scope and contributions of this research

The main objective of this study is to develop a robust multiresponse structural parameter estimation method for the automated FE model updating. The proposed method is verified using data obtained from a set of nondestructive tests conducted on the UCF grid. In the present work, experimental data quality criteria was established and implemented to reduce the measurement errors in the model calibration process. To lessen modeling error, an accurate FE model was created with all the details including geometry, section properties, and boundary conditions. Error sensitivity analysis was performed using simulated damage cases to estimate the parameters with least sensitivity to measurement errors and also to determine the most observable parameters in presence of measurement errors. In these simulations, various grouping of elements with the same properties were used to reduce the number of unknowns, and then ungrouping was utilized to reduce the modeling errors. In this process a set of observable and error tolerant unknown parameter groups was selected for the purpose of FE model calibration using experimental data. Multiresponse parameter estimation based on the static and modal data collected from the UCF grid was used to calibrate the finite element model such that it can reflect the actual behavior of the structure more closely. The modal data includes the natural frequencies and the corresponding mode shapes whereas the static data includes displacement, rotation and strain, which have been used in this study. Selecting an observable and error tolerant set of unknown structural parameters and using the best subset of measured data led to successful simultaneous estimation of stiffness and mass parameters. To overcome the probable ill-conditioning in solving system of equations, which can result in poor parameter estimation, four different types of normalizations were applied at various levels of the proposed method. Using these normalizations enabled the usage of multiresponse parameter estimation method based on static and modal experimental data. A novel statistical normalization method based on the quality of the measured static data, instead of only applying weight factors, was used to provide a robust parameter estimation methodology.

The major contributions of this research presented for the first time are: (1) use of measured static and dynamic data for multiresponse parameter estimation, (2) implementation of real-time communication between two separate commercial software packages for automated finite element model updating, (3) development of NDT data quality analysis to reduce measurement errors, (4) implementation of a novel statistical normalization method based on the quality of the measured static data for use in parameter estimation methodology, (5) use of Monte Carlo analysis to...
identify error sensitive parameters, (6) estimation of a set of most observable and error tolerant parameters for a laboratory model, and (7) creation of a platform that can be utilized for full scale structural model updating.

1.2. Software platforms used in this research

The present study is based on the parameter estimation formulations developed by researchers at Tufts University that are used in PARIS© (PARameter Identification System) for automated FE model calibration of full scale structures. PARIS is a custom MATLAB® [22] based computer program which uses SAP2000® [23] as a slave FE analysis engine using Application Programming Interface (API) for finite element analysis (FEA). APIs allow real-time exchange of information between MATLAB and SAP2000 during the iterative stages of FE and updating model parameters in the optimization process, which makes this program more applicable to large structures in comparison with its previous versions. PARIS is designed for calibrating FE models comprised of frame, quadrilateral planar shell, and cuboid solid elements using static and/or modal data. Automated FE model updating in PARIS is based on minimizing the residual between the predicted response of the FE model and the measured data from the NDT. In the parameter estimation process of the UCF grid that was based on experimental data, both mass-based and flexibility-based error functions have been used. PARIS is open source research software and has been posted at the SHM research website of the Civil and Environmental Engineering Department at Tufts University at http://engineering.tufts.edu/cee/shm/software.asp.

2. Error functions and scalar objective function

In the parameter estimation process, error functions play a pivotal role. In general, error functions define the difference between analytical and experimental measurements. The estimation of an unknown parameter is the result of finding the global minimum of the error function. Error functions represent a residual between the predicted responses from the computer model, such as finite element model (FEM) and the experimental data. Error functions are generally defined as the residuals between predicted and measured system responses, in which \( q \) represents the response quantity, \( e(p) \) is the error function, and \( p \) represent the unknown structural parameters.

\[
e(p) = q_{\text{predicted}} - q_{\text{measured}}
\]

The response of the structure can be quantified under a set of known static loads or modal excitations that categorize error functions as Static-based and Modal-based. To find the discrepancy between the analytical and experimental response, the main categories of error functions have been subdivided into stiffness and flexibility-based error functions. The stiffness-based error function measures the residual between applied forces, and the flexibility-based error function determines the difference between experimental and analytical displacements estimation of the unknown structural parameters. These error functions are implicitly a function of the structural parameters \( p \). Examples of the unknown parameters include axial rigidity (EA), bending rigidity (EI) and torsional rigidity (GJ) for frame elements. For plates and shells, modulus of elasticity and plate thickness are used, and for isoperimetric solids, the modulus of elasticity is used as unknown parameters. For dynamic systems, lumped mass and/or distributed mass can be used as modeled in the FE model. Link elements are also used to represent axial, bending and torsional springs in 2D and 3D spaces.

2.1. Static flexibility (SF)

The static flexibility-based error function was developed by Sanayei et al. [24] and is based on the residual between measured and analytical displacements only for measured responses.

\[
[e_{SF}(p)] = [K(p)]^{-1} [F] - [U]
\]

where \([F]\) and \([U]\) represent the applied live loads cases and the corresponding measured displacements per load case. The error function \( e_{SF} \) is calculated based on the analytical stiffness matrix \([K(p)]\) and known applied live load cases.

2.2. Static strain (SSTR)

Sanayei and Saletnik [25] developed a strain-based error function that is the residual of predicted and measured strains at selected observation points. The B matrix was used to map the nodal displacements to strains along the length of each member.

\[
[e_{SSTR}(p)] = [B][K(p)]^{-1}[F] - [\varepsilon]
\]

In Eq. (3), \([\varepsilon]\) is the measured strain matrix for load cases.

2.3. Modal flexibility (MF)

Sanayei et al. [26] developed the modal flexibility-based error function that is the residual between analytical and measured modal displacements. The inverse form of the analytical stiffness matrix \([K(p)]\) was used in the formulation of the modal-flexibility error function and analytical mass matrix \([M(p)]\).

\[
[e_{MF}(p)] = \omega^2 [K(p)]^{-1} [M(p)] \{ \phi \} - \{ \phi \}
\]

Mode shapes \( \{ \phi \} \) and natural frequencies \( \{ \omega \} \) can be used as the input data based on the identified modal parameters from the experimental data.

2.4. Scalar objective function

The scalar objective function \( J(p) \) is created in Eq. (5). The value of \( J(p) \) is minimized during the parameter estimation process using numerical sensitivities. In this work, the constrained nonlinear multivariable function \( fmincon \), which is available in MATLAB Optimization Toolbox, was used as the minimization technique. When neither the modeling error nor the measurement error is a part of the parameter estimation, the scalar objective function will approach zero.

\[
J(p) = \sum_i \sum_j |e_i(p)|^2
\]

where \( i \) represents a measurement at a selected DOF and \( j \) represents the number of sets of forces.

To verify the final result of the \( J(p) \) minimization, which should have resulted in the global minimum, multiple initial starting points were selected. Since the final results from multiple initial points were converged on the unique answer, it was concluded with high possibility that the global minimum had been reached in the minimization procedure.

3. Normalization techniques, parameter grouping and multiresponse parameter estimation

In solving any system of equations, there is the possibility of facing ill-conditioning resulting in poor estimation of the unknown parameters; therefore four types of normalizations were proposed to prevent such numerical challenges while using a combination of error functions with different types of measurements.
3.1. Parameter normalization

In normalization at the parameter level, the estimated value was divided by the initial value of the parameter. Ratios less than one indicate degradation in the value of the parameter in comparison with the initial guess, and ratios larger than one represent that the actual value of the parameter is larger than the initial estimate.

3.2. Error function normalization

To ensure that all the measurements have a reasonable influence on the estimation process, a normalization technique was needed to apply to the error functions, while multiple sets of measurements, each in different units and scales, were used in the parameter estimation procedure. In the case of combining multiple measurements without error function normalization, the final result could not be robust since measurements with different magnitudes were used in the scalar objective function. The larger measurements could potentially overshadow the smaller measurements during the parameter estimation process. For instance, displacements were measured on the order of millimeters \((10^{-3})\), while strains were measured on the order of micro strains \((10^{-6})\). In order to overcome this issue, each error function was divided by its initial value as the optimization iterations progress. This method was used to make the error functions normalized and unitless to reduce ill-conditioning in the system of equations.

3.3. Objective function statistical normalization

In the normalization method presented by DiCarlo [27], the static-based error functions were normalized with respect to their standard deviations, which make the objective function weighted, normalized and unitless. This method also resolved the different order of magnitudes issue for multiple measurement types. When objective function normalization was applied, each measurement was weighted proportional to the inverse of its uncertainty in the estimation procedure.

\[
f(p) = e(p)^T \Sigma_e^{-1} e(p)
\]  

In Eq. (6), \(\Sigma_e\) is the covariance matrix of the measured response data. To find the parameter that makes the measured data the “most probable,” the maximum likelihood estimator was used. Using this normalization technique was also a very efficient approach to smooth the objective function surface.

3.4. Objective function weighting

In using multiresponse parameter estimation, to prevent the domination of a scalar objective function by another one, there is a need to normalize the objective functions. Each objective function was normalized with respect to their initial value, rendering a weight of 1.0 to each objective function. If needed, each scalar objective function can be multiplied by a suitable weight factor based on the measurement types and the number of measurements used.

3.5. Parameter grouping

Parameter grouping is a technique that allows assigning parameters with the same structural parameters (e.g., mass, stiffness and area) into one group. Grouping reduces the total number of unknown parameters, which results in fewer of the required measurements for the parameter estimation process.

3.6. Multiresponse parameter estimation

Multiresponse parameter estimation is an effective method by which to simultaneously use different error functions that are based on different types of measurements, which allows the combination of static and modal data in the model updating procedure. Using error function stacking would increase the number of measurements in the parameter estimation process that can result in estimating unknown parameters. To have the information of each error function in this technique, the vector of \(e_{\text{Stack}}(p)\) has been created as follows:

\[
e_{\text{Stack}}(p) = \begin{bmatrix} e(p)_1 \\ e(p)_2 \\ \vdots \\ e(p)_n \end{bmatrix}
\]

where \(n\) = number of error functions.

4. University of Central Florida benchmark

The University of Central Florida (UCF) benchmark is a scale bridge model that was designed as a grid system. It was designed, constructed and used for experimental studies at UCF, see Fig. 1. As stated in Catbas et al. [18], this model is a multi-purpose specimen, enabling researchers to use it as a test bed for different techniques, algorithms and methods of the model updating procedure. The UCF grid has two longitudinal girders which run continuously for the whole length of the structure, each with a length of 5.48 m (18 ft), with two clear spans and seven transverse beams connecting the two girders. As stated in Burkett [28], the transverse beams are connected into girders by two angle clips and two cover plates to provide shear and moment transfer, which makes it a partially restrained connection, see Fig. 2. Both girder and beams are A36 steel S3x5.7 standard sections. These sections were found to be the best fit to make the modal frequencies, deflections and slopes represent the short to medium span bridges. To reduce the effect of support vertical movements, W12x28 sections were used as piers, each with a length of 1.07 m (42 in.) and fixed at the base.

The UCF benchmark has complex boundary conditions as shown in Fig. 2. The pier to girder connection was designed to behave as an ideal roller support. It consists of a cylinder sandwiched between two curved bearing plates which allow the translation and rotation in the plane of bending for the girders.

4.1. Instrumentation plan

For static testing, the structure was instrumented with 8 vibrating wire strain gages (Geokon model 4000), 4 vibrating wire...
tiltmeters (Geokon model 6350) and 4 displacement gages (Space Age Control). Fig. 3 shows the instrumentation details and sensor numbering for static testing. Displacement gages (DG) were connected between the floor and underneath four of the connections. Tiltmeters (TM) were mounted on the longitudinal members with the typical distance between each end of the girders and the closest tiltmeter (TM) was 178 mm (7 in.). Strain gages (SG) were welded to the top and/or bottom flanges of the longitudinal members and placed in the 266.7 mm (10.5 in.) from the closest transverse beam.

Eight accelerometers (PCB 393C) were used for dynamic testing on the UCF grid. Each accelerometer was placed vertically and the grid was excited at four different locations. Fig. 4 shows the accelerometer and excitation locations for dynamic testing using an impact hammer (IPC 086D20). Based on the instrumentation of the structure, a multi-input multi-output data set with 4 sets of impact forces and 8 sets of acceleration time histories was created.

4.2. Finite element model for the UCF benchmark

A finite element model was created in SAP2000 based on the design calculations and CAD drawings in Burkett [28]. Table 1 shows the initial section properties for the finite element model. Fig. 5 shows the finite element model of the UCF benchmark. All of the structure’s elements were modeled as A36 steel with a Young’s modulus of 200 GPa (29,000 ksi). For the girder and beams, S3×5.7 standard sections have been used, and for supports, W 12 × 26 sections have been used as piers, each with a length of 1.07 m (42 in.) and fixed at the base. Section properties for these standard sections can be found easily in the AISC manual [29]. The longitudinal, transverse, and support members are modeled as frame elements with their respective structural properties based on structural shape dimensions. The support columns are assumed to have fixed boundary conditions at the base. To have the best a priori model for the purpose of finite element model updating, the authors tried to create the geometry of the UCF grid FE model as accurately as possible. To achieve this goal, the support columns have been extended to the center of the rotation of roller supports, and the connections between the grid and the support members were modeled using vertical link elements; the six stiffness values of the boundary links can be adjusted to accurately represent the boundary conditions.

4.3. Boundary stiffness sensitivity study

To determine the range for the boundary link elements in the UCF grid, Sipple [30] performed a sensitivity study. Boundary
and stiffness direction was used since gravity for the active boundary stiffness directions, which are and fixed boundary conditions. This study was only performed condition stiffness has been divided into free, partially restrained and fixed boundary conditions. This study was only performed for the active boundary stiffness directions, which are $K_y$, $K_z$ and $K_{hp}$. Table 2 shows the boundary stiffness ranges.

In this work, only $K_y$ stiffness direction was used since gravity loading and data acquisition was performed only in the vertical direction.

### 4.4. Nondestructive testing of UCF benchmark

Nondestructive tests were designed and performed on the UCF grid by applying static gravity loadings and dynamic loadings using an impact hammer. For static testing, vertical displacements, rotations about y-axis, and longitudinal strains on beams were measured. Three identical static NDTs were performed on the UCF grid using 8 different load cases, and static responses were measured.

Dynamic tests were used to determine the modal parameters of the UCF grid, i.e., frequencies and mode shapes. An impact hammer was used to excite 4 nodes, and time history accelerations were measured with all 8 accelerometers. The modal measured data were processed at UCF by Catbas et al. [18] using the Complex Mode Indicator Function (CMIF) algorithm to identify modal parameters, such as natural frequencies and mode shapes of the structure.

### 4.5. Static load test

The main rationale for the measurement and loading locations was to stress all the main structural elements that affect the characteristic deflections of the system. To mitigate uplift at the supports due to individual live loads, an additional dead weight of 178 N (40 lb) was permanently placed on the grid at all support points during the static tests. There were a total of 8 load cases for static testing; each of them includes 2 or 4 point loads, which were about 670 N (150 lb). Table 3 shows the location of each load case. The structure was tested three times independently under each load case which involved unloading and reloading the structure for each measurement. Ideally, re-instrumenting the structure for each trial would have been required to have the true independency of measurements; however, since this would have taken a prohibitively long time, it was not considered.

---

**Table 1**

<table>
<thead>
<tr>
<th>Model element</th>
<th>AISC classification</th>
<th>$A_x$ [m² (ft²)]</th>
<th>$I_x$ [m⁴ (ft⁴)]</th>
<th>$I_y$ [m⁴ (ft⁴)]</th>
<th>$E I_x$ [N m² (lb ft²)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>S3x5.7</td>
<td>$1077 \times 10^6$</td>
<td>$1.05 \times 10^6$</td>
<td>$0.19 \times 10^6$</td>
<td>$20.98 \times 10^6$</td>
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<td>(115.92 \times 10^4)</td>
<td>(115.92 \times 10^4)</td>
<td>(115.92 \times 10^4)</td>
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<td>(115.92 \times 10^4)</td>
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<tr>
<td>Beam</td>
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<td>$0.19 \times 10^6$</td>
<td>$20.98 \times 10^6$</td>
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<td>(115.92 \times 10^4)</td>
<td>(115.92 \times 10^4)</td>
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<td>(115.92 \times 10^4)</td>
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<td>Girder connection</td>
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**Table 2**

<table>
<thead>
<tr>
<th>Stiffness direction</th>
<th>Free</th>
<th>Partially restrained</th>
<th>Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_v$ [N/m (Kip/in)]</td>
<td>$0-2 \times 10^5$ (114)</td>
<td>$2 \times 10^5$ (114)-$2 \times 10^6$ (11,420)</td>
<td>$2 \times 10^6$ (11,420)</td>
</tr>
<tr>
<td>$K_x$ [N/m (Kip/in)]</td>
<td>$0-5 \times 10^5$ (228)</td>
<td>$5 \times 10^5$ (228) to $2 \times 10^6$ (11,420)</td>
<td>$2 \times 10^6$ (11,420)</td>
</tr>
<tr>
<td>$K_y$ [rad/m (rad/in)]</td>
<td>$0-4 \times 10^5$ (10)</td>
<td>$4 \times 10^5$ (10)-$4 \times 10^6$ (10⁶)</td>
<td>$4 \times 10^6$ (10⁶)</td>
</tr>
</tbody>
</table>

**Fig. 5.** UCF benchmark finite element model.

*Image with UCF grid and section properties.*
More observations were performed. The accumulation of the final estimations became more stable and accurate as the statistical properties of parameter estimates, such as mean and standard deviation, were calculated. The mean and standard deviation of parameter estimates were saved. Using all Monte Carlo simulations, statistical analysis of a complex system in which numerous mathematical operations create a nonlinear system of equations. Using Monte Carlo simulations can determine how error in measurements can affect the parameter estimation process, since accuracy of parameter estimates is one of the essential components for a successful finite element model calibration. This method was used by selecting a large number of observations (NOBS) and performing an independent parameter estimation for each of them in the presence of simulated measurement error. When parameter estimation was completed for each observation, the final contaminated parameter estimates were saved. Using all Monte Carlo simulations, statistical properties of parameter estimates, such as mean and standard deviation, were calculated. The mean and standard deviation of the final estimations became more stable and accurate as more observations were performed.

### 4.6. Dynamic impact test

The dynamic test plan for the UCF grid consisted of four vertical impact locations with 8 vertical acceleration response measurement locations. The test consisted of hitting the vertical modes of the structure because these could have given more information about the strong axis stiffness of each member, similar to the static tests under gravity loadings. During the instrumentation process of the UCF grid, it was believed that measuring the vibrations of the middle connections in comparison with the connections above the piers would have resulted in the better characteristic mode shape of the scale bridge model; this was because there was not enough vibration to measure with accelerometers since the vertical direction was restrained by the model supports.

### 5. Error sensitivity analysis using Monte Carlo simulations

Monte Carlo (MC) analysis is a powerful tool for performing statistical analysis of a complex system in which numerous mathematical operations create a nonlinear system of equations. Using Monte Carlo simulations can determine how error in measurements can affect the parameter estimation process, since accuracy of parameter estimates is one of the essential components for a successful finite element model calibration. This method was used by selecting a large number of observations (NOBS) and performing an independent parameter estimation for each of them in the presence of simulated measurement error. When parameter estimation was completed for each observation, the final contaminated parameter estimates were saved. Using all Monte Carlo simulations, statistical properties of parameter estimates, such as mean and standard deviation, were calculated. The mean and standard deviation of the final estimations became more stable and accurate as more observations were performed.

### 5.1. Error sensitivity analysis for the UCF benchmark

The purpose of the error sensitivity analysis was to determine the effect of measurement error in the parameter estimation process of the UCF grid. In each case of Monte Carlo simulation, 50 observations (NOBS) were chosen for different simulated parameter estimation scenarios. Various numbers of MC experiments were performed and it was determined the 50 observations were sufficient to produce reasonable input-output error estimates. In this process, unknown mass and stiffness parameters were identified in different simulated scenarios. In each simulated scenario, all of the structural parameters except the damage ones were assumed to be known with a high level of confidence. The number and location of the measured responses were selected as it was in the non-destructive test of the UCF grid. The response data were simulated to have a uniform-proportional random error in the range of 5%. This range was believed to provide a reasonable and practical level of measurement error. In order to find the influence of the measurement error based on different sensors and the respective error function, stacking was not used in Monte Carlo simulations.

To find the strong axis bending rigidity ($E I_z$) and mass of the main structural members and connection members, three member groups were defined: (G1) L shape corner connections, (G2) T shape middle connections and (G3) transverse beams and longitudinal girders. Fig. 6 shows the parameter grouping scheme which was used in all parameter estimations. Both the stiffness and mass parameter values for all the boundary links in the vertical direction (z-direction) were grouped, creating six parameter groups for the UCF grid error sensitivity analysis.

The parameter grouping scheme was designed based on the results of the Monte Carlo simulations for multiresponse parameter estimation using experimental data. All excitations and sensors were applied to longitudinal members 1 and 3 and none on the transverse members A through G. According to the error sensitivity analysis for the given loadings and sensor locations, both T and L shape connections should be grouped as one for better observability and estimation of connections stiffness and mass parameters. It was also necessary for the longitudinal and transverse beams to be grouped together for stiffness and mass parameter estimations. Given the current set of measured excitation and response locations, this study resulted in forming five highly observable and error tolerant parameter groups consisting of stiffness and mass of all beam members and stiffness and mass of all connections along with the boundary link stiffness. Rolled beam section properties are known with a high confidence. Their stiffness and mass properties are kept as unknown for quality control of the unknown connection stiffness and mass parameters. Based on MC simulation results, feasible parameter estimation cases to be used with the experimental data from the UCF grid were determined.

![Parameter grouping scheme for UCF benchmark](image-url)
6. Parameter estimation of the UCF benchmark using experimental data

Since the precision of the measurements has a major impact on the final results of the model updating procedure, data quality analysis was performed on the measured data from nondestructive tests. The goal of this analysis was to minimize the influence of the measurement noise by finding the best subset of the measured data and dropping the noisy measurements before using them in the parameter estimation process. In general, measurement error is the difference between actual and measured values in measurements. The source of the noise in measurements can be from electronic devices (e.g., sensor types, signal conditioning, electronic random noise picked up by unshielded wires), installation faults and/or ambient noise. In addition, ambient vibrations are always present, adding to the signal noise.

6.1. Experimental data quality study

Based on the level of noise to signal ratio and confidence in the measured values, some measurements were eliminated from the measured test dataset. To select the noisiest measurements, standard deviation of measured data in the unloaded and loaded phases were calculated. The larger values of the standard deviation of the measured data were used for selecting the noisiest sensors. For each sensor, values smaller than ten times of standard deviation of the measurements were eliminated from the dataset. For example, the static strain measurements were all in the order of 0–60 με, and since the standard deviation of data in most strain gages were around 0.5 με, it was decided to eliminate the measurements that were smaller than 5 με. Eliminating the measured values based on the noise floor was also applied to measured displacements and slopes which ensured that smaller measured values were eliminated since they were mainly delivering more noise to the objective function. For example, slope values less than 0.01° were eliminated since it has been assumed to be contaminated by a high level of error; slopes were in the order of 0–0.06°. Displacements were in the order of 0–0.762 mm (0.03 in), where measurements less than 0.254 mm (0.01 in.) were eliminated based on the noise floor.

Since three trials for each measurement set were conducted, one other criterion was to use only the values that are based on exactly three independent tests and no less. The reason for this was to have consistency in the level of confidence in the measured data values. The average value of three sets of tests was used in the parameter estimation process.

Table 4 presents the final outcome of the data quality analysis of the measured responses from the static test indicated by the sensor numbers. These data were used in the final parameter estimation process of the UCF grid.

All the modal data delivered from the UCF grid was also examined for reliability. Experimental mode shapes were compared to analytical mode shapes using the Modal Assurance Criterion (MAC). Allemang [31] presented the following formulation for MAC:

\[ \text{MAC}_{ij} = \frac{(\phi_i^e \phi_j^a)^2}{(\phi_i^e \phi_i^e)(\phi_j^a \phi_j^a)} \]  

In Eq (8), \( \phi_i^e \) is the ith experimental mode shape and \( \phi_j^a \) is the jth analytical mode shape.

MAC values depicts the degree of fitness between the experimental and analytical models. Diagonal values close to 1.0 indicate a close correlation between these two models. The nonzero off-diagonal terms indicate some degree of correlation between the corresponding modes. For visualization purposes, values higher than 0.1 are shown in bold. Since the number of measurement points used to determine the mode shapes was a small subset of the available degrees of freedom, MAC values cannot perfectly show the correlation between the analytical and experimental mode shapes. Considering the inaccuracy in the measured modal responses in higher frequencies and the calculated MAC values (Table 5), only the first 5 mode shapes were used for calibration of the UCF grid model.

6.2. Multiresponse parameter estimation and model calibration of the UCF benchmark

The scope of the parameter estimation of the UCF grid was to update both mass and stiffness properties. Parameters were selected for the estimation process due to their observability. These parameters were: (a) Strong axis bending rigidity (\( E_1 \)), (b) Area mass (\( A_m \)), and (c) Boundary link stiffness \( K_z \). \( E_1 \) and \( A_m \) were estimated for both main structural members and connection members. Model updating was not performed on parameters such as weak axis bending rigidity and shear rigidity because of their low or no observability. Boundary link elements that were not observable, such as rotation in all three directions and translation in x and y directions, were also not included as unknown parameters.

Table 4
Sensors numbers used for measured static responses.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Disp. gage</th>
<th>Tiltmeter</th>
<th>Strain gage</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC 1</td>
<td>2, 4</td>
<td>1, 2, 3, 4</td>
<td>1, 3, 4, 5, 6, 7</td>
</tr>
<tr>
<td>LC 2</td>
<td>1, 2</td>
<td>1, 2</td>
<td>1, 3, 4</td>
</tr>
<tr>
<td>LC 3</td>
<td>4</td>
<td>3, 4</td>
<td>5, 6, 7</td>
</tr>
<tr>
<td>LC 4</td>
<td>2, 4</td>
<td>1, 2, 3, 4</td>
<td>1, 2, 3, 4, 5, 7</td>
</tr>
<tr>
<td>LC 5</td>
<td>1, 2, 3</td>
<td>1, 2, 3, 4</td>
<td>1, 3, 4, 5, 7</td>
</tr>
<tr>
<td>LC 6</td>
<td>1, 2, 3, 4</td>
<td>1, 2, 3, 4</td>
<td>1, 3, 4, 5, 7</td>
</tr>
<tr>
<td>LC 7</td>
<td>1, 2, 3</td>
<td>1, 2, 3, 4</td>
<td>1, 2, 3, 4, 5, 7, 8</td>
</tr>
<tr>
<td>LC 8</td>
<td>2, 4</td>
<td>1, 2, 3, 4</td>
<td>1, 3, 4, 5, 7, 8</td>
</tr>
<tr>
<td>Total data points</td>
<td>19 of 32</td>
<td>28 of 32</td>
<td>42 of 64</td>
</tr>
</tbody>
</table>

Table 5
MAC values using initial parameters analytical modes.

<table>
<thead>
<tr>
<th>Modes</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
<th>A7</th>
<th>A8</th>
<th>A9</th>
<th>A10</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.998</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.065</td>
<td>0.003</td>
<td>0.001</td>
<td>0.986</td>
<td>0.011</td>
<td>0.000</td>
</tr>
<tr>
<td>E2</td>
<td>0.000</td>
<td>0.995</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E3</td>
<td>0.000</td>
<td>0.000</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
<td>0.151</td>
<td>0.004</td>
<td>0.001</td>
<td>0.013</td>
<td>0.988</td>
</tr>
<tr>
<td>E4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.995</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E7</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.150</td>
<td>0.000</td>
<td>0.000</td>
<td>0.033</td>
<td>0.000</td>
</tr>
<tr>
<td>E8</td>
<td>0.997</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.070</td>
<td>0.002</td>
<td>0.000</td>
<td>0.983</td>
<td>0.014</td>
</tr>
<tr>
<td>E9</td>
<td>0.001</td>
<td>0.000</td>
<td>0.997</td>
<td>0.001</td>
<td>0.144</td>
<td>0.004</td>
<td>0.001</td>
<td>0.020</td>
<td>0.979</td>
<td>0.000</td>
</tr>
<tr>
<td>E10</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
The results of the model calibration are shown in Table 6. In this table, estimated bending rigidity (EIz) of elements about their strong axis, and the cross-sectional area of elements (Am) used for mass calculation in dynamic analysis, were presented as the normalized values with respect to their initial guesses. Blank cells denote “not applicable” because the static error function is not for estimation of mass parameters. All initial conditions for longitudinal, transverse, and connection groups are assigned as the properties of S3x5.7 presented in Table 1. The initial Kz value for the vertical supports’ spring stiffnesses in all cases was 2.1 × 10^9 N/m (12,000 Kip/in). In the experimental grid nondestructive tests, the static loads were applied in the vertical direction (gravity loadings) and all dynamic impacts were applied in the vertical direction. In the process of parameter estimation using the static and modal data, seven combinations of error functions were used to examine their capabilities individually and together for multiresponse parameter estimation in four different parameter estimation cases.

Column 1 of Table 6 shows the error function combinations used. In each case the parameter estimation was performed simultaneously for all the unknown parameters shown in column 2. Other parameters not included in column 2 were assumed to be known in each case using the initial parameter values. Parameter grouping was also applied to all of the structural parameters using the same modifier. Estimated values in column 3, 4 and 6, demonstrate the usage of error functions SF, SSTR and MF individually. The rest present use of multiresponse parameter estimation using more than one set of measured data with the associated error functions.

In Case 1, the EIz parameter of the main structural members and connection members were estimated. Due to complex shape of connection stiffener plates and angle clips with bolt holes, their effective bending rigidities were unknown. As a result, EIz of the standard beam sections were used as the initial value for both the beams and connections. All cases using individual error functions or multiresponse error functions converged. Overall, the estimated bending rigidities for beam sections were close to the values of the standard section. The values for connection bending rigidities were much larger than the beam values. These larger values are reasonable since the connections were built up with plates connecting the flanges and webs of the longitudinal and transverse members shown in Fig. 2. The parameter estimates improved as more data was added, especially modal data, using multiresponse parameter estimation.

In Case 2, rigidities and mass parameters of beams and connections of the UCF grid were estimated. It was observed that by adding measured modal data to the estimation process, there was more consistency in the final parameter estimates. Since mass parameters were involved, the static error functions were not applicable.

In Case 3, there was interest to determine if there was any vertical flexibility due to floor, column, roller, etc. For this purpose, the Ki stiffness values for all boundary links were considered as one unknown group. Four groups of parameters from members and connections stiffness and mass properties, plus one vertical boundary stiffness group, resulted in five unknowns for this case.

In Case 4, the total of ten unknown parameters, including four groups of member stiffness and mass, added to six individual Ki values for estimation. To reduce the effect of modeling on parameter estimates, all significant structural parameters engaged in resisting vertical deformations were assumed to be unknown in cases 3 and 4. Based on the estimated parameters in the last two cases, there was no real change in stiffness and mass parameters of the longitudinal girders and transverse beams.

In summary, there was an increase of about 80% in bending rigidity and 50% in mass of the connections. This was expected since cover plates, angle clips, bolt holes, and bolts were used in these connections. Boundary link stiffness Kz remained unchanged in the range of fully fixed, based on Table 2. Estimated bending rigidity of the connection members in Case 1 had larger values in comparison with other cases. Based on the error sensitivity analysis performed for SF and SSTR error functions individually, it was determined that estimation of bending rigidity parameter (EIz) for longitudinal and transverse beams was more error-tolerant than connection members. MC simulations were showed larger estimated values for EIz of connection members than the true values. Since mass parameters were assumed to be known in Case 1, due to a modeling error caused by this assumption, it was predictable to have errors in finding the stiffness parameters alone.

**Table 6**

<table>
<thead>
<tr>
<th>Case</th>
<th>Estimated parameters</th>
<th>SF</th>
<th>SSTR</th>
<th>SSTR + SF</th>
<th>MF</th>
<th>MF + SF</th>
<th>MF + SSTR</th>
<th>MF + SF + SSTR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EIz Long &amp; Trans Beams</td>
<td>1.01</td>
<td>0.98</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>EIz connection</td>
<td>2.09</td>
<td>1.75</td>
<td>1.84</td>
<td>1.88</td>
<td>1.88</td>
<td>1.88</td>
<td>1.88</td>
</tr>
<tr>
<td>2</td>
<td>EIz long &amp; trans beams</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>EIz connection</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.78</td>
<td>1.79</td>
<td>1.80</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>Am long &amp; trans</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Am connection</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.50</td>
<td>1.50</td>
<td>1.51</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>(grouped)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.45</td>
<td>1.49</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>3</td>
<td>EIz long &amp; trans beams</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>EIz connection</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.78</td>
<td>1.80</td>
<td>1.78</td>
<td>1.79</td>
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<tr>
<td></td>
<td>Am long &amp; trans</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td></td>
<td>Am connection</td>
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<td>–</td>
<td>1.51</td>
<td>1.51</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>K1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.49</td>
<td>1.46</td>
<td>1.46</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>K2</td>
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<td>–</td>
<td>–</td>
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<td>1.50</td>
<td>1.46</td>
<td>1.50</td>
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<tr>
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<td>K3</td>
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<td>–</td>
<td>–</td>
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<td>1.59</td>
<td>1.47</td>
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<tr>
<td></td>
<td>K4</td>
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<td>–</td>
<td>1.50</td>
<td>1.65</td>
<td>1.47</td>
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<td>K5</td>
<td>–</td>
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<td>–</td>
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<td>1.15</td>
<td>1.45</td>
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</tr>
<tr>
<td></td>
<td>K6</td>
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<td>–</td>
<td>–</td>
<td>1.48</td>
<td>1.28</td>
<td>1.45</td>
<td>1.20</td>
</tr>
</tbody>
</table>

6.3. FE model on pins and rollers used in multiresponse parameter estimation and model calibration

According to the estimated values of Kz stiffness presented in Table 6, and in comparison with the boundary stiffness ranges in Table 2, it was realized that the Kz values were all in the “fixed”
range. Based on this, it has been decided to create a new FE model with “Pin-Roller-Roller” boundary conditions. The boundaries at A1 and A3 were assumed to be pin supports and the rest were modeled as roller supports. The parameter estimation was also performed using the pins and rollers FE model and the same experimental static and modal data from the UCF grid. The values that were presented in Table 1 were used to define the $E_I$ and $A_m$ parameter’s initial values. The parameter estimates are summarized in Table 7. The estimates for the pins and rollers boundary conditions presented in Table 7 closely matched with the estimates in Table 6 that were based on the previous larger model. The small differences in parameter estimates are due to using a different size FE model whose sensitivities to measurement and modeling errors can be different.

MAC matrix presented in Table 8 shows an excellent match between the first 6 mode shapes of the calibrated model and measured data based on the Case 2 of Table 7 using MF, SF and SSTR error functions simultaneously. From these results it is inferred that the pins and rollers model is a better representation of the measured data.

To verify the estimated values and assess the physical meaning of the parameters, a direct comparison with the experimental data provides an objective and quantifiable basis for comparison. The predicted responses based on the calibrated model in Case 2 (with 4 unknown parameters) were compared with experimental responses. Both the static and modal responses for the updated models were compared with the experimental data. Displacements, slopes, strains and mode shapes were graphed to compare the predicted response from the calibrated model with the measured response from the nondestructive tests. It is important to remember that static displacements, slopes, and strains are impacted only by the stiffness parameters and not by the mass parameters. Both stiffness and mass parameters affect modal frequencies and mode shapes of a structure that present the global model calibration.

Figs. 7 and 8 show typical strains from LC1 and LC4 for 6 gages on the grid, which were used in the parameter estimation of the UCF Grid and show the analytical prediction of the uncalibrated and calibrated models for comparison with the measured strains. These plots show an excellent match between the calibrated model response and experimental measurements.

Figs. 9 and 10 show typical displacements from 2 gages along longitudinal girders (one per girder) for LC5, showing the analytical prediction of the uncalibrated and calibrated models for comparison with the measured displacements. These plots show an improvement to the match between the calibrated model response and experimental measurements. Although the predicted responses moved toward the measurements, some of them did not match with the measured displacements as well as other

Table 7
UCF benchmark parameter estimates based on the pins and rollers model.

<table>
<thead>
<tr>
<th>Case</th>
<th>Estimated parameters</th>
<th>SF</th>
<th>SSTR</th>
<th>SSTR + SF</th>
<th>MF</th>
<th>MF + SF</th>
<th>MF + SSTR</th>
<th>MF + SF + SSTR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E_I$, long &amp; trans beams</td>
<td>1.02</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$E_I$, connection</td>
<td>1.82</td>
<td>1.79</td>
<td>1.85</td>
<td>1.88</td>
<td>1.88</td>
<td>1.88</td>
<td>1.88</td>
</tr>
<tr>
<td>2</td>
<td>$E_I$, long &amp; trans beams</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
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</tr>
<tr>
<td></td>
<td>$E_I$, connection</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.78</td>
<td>1.81</td>
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<td></td>
<td>$A_m$, long &amp; trans</td>
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<td>-</td>
<td>-</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$A_m$, connection</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 8
MAC values for calibrated model on pins and rollers analytical modes.

<table>
<thead>
<tr>
<th>Modes</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
<th>A7</th>
<th>A8</th>
<th>A9</th>
<th>A10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>0.998</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.498</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E2</td>
<td>0.000</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E3</td>
<td>0.000</td>
<td>0.000</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.999</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E5</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.998</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.005</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E7</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
<td>0.159</td>
<td>0.000</td>
<td>0.000</td>
<td>0.035</td>
<td>0.000</td>
<td>0.003</td>
<td>0.036</td>
</tr>
<tr>
<td>E8</td>
<td>0.997</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.487</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>E9</td>
<td>0.002</td>
<td>0.000</td>
<td>0.996</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>E10</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
displacements did. This might be due to errors in displacement measurements.

Figs. 11 and 12 show typical slopes from LC1 and LC6 for 4 tiltmeters on the grid which were used in the parameter estimation of the UCF Grid and show the analytical prediction of the uncalibrated and calibrated models for comparison with the measured strains. These plots show an excellent match between the calibrated model response and experimental data.

Along with the direct comparison between the predicted and experimental responses of the UCF grid, Root Mean Square (RMS) values for the predicted and experimental measurements were also calculated in all load cases for each type of measurements. Table 9 presents the RMS values.

RMS values of the predicted responses were calculated for both calibrated and uncalibrated FE models. Percentage error of the predicted RMS responses with respect to the experimental is data also presented in Table 9. The RMS percentage errors show great improvements for all three types of predicted responses, more so for strains and slopes. This might have to do with lower noise to signal ratios in measured strains and slopes. The UCF grid model supported on pins and roller has a slightly lower average percentage error compared to the model supported on link elements (springs). Overall, both of the calibrated models with measured experimental data showed great improvements in predicting responses compared to the uncalibrated model.

Fig. 13 shows the five natural frequencies used in parameter estimation. Generally all natural frequencies moved toward the measured data. Frequencies of the calibrated model show a great match with the measured frequencies in modes 1, 2, 3 and 5.

Table 10 presents the comparison of the mode shapes and frequencies of the UCF Grid. It compares the uncalibrated and calibrated FE model modal data with the measured data for the first 5 mode shapes of the UCF grid. Since each mode shape used only 8 measured degrees of freedom, it is believed that the most efficient observability occurred only in the first few modes.

Percentage errors and the average errors of the predicted natural frequencies of the uncalibrated and calibrated FE models were

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**Table 9**

Predicted vs. experimental RMS responses.

<table>
<thead>
<tr>
<th>Measurement type</th>
<th>Uncalibrated FE model on link support RMS</th>
<th>Calibrated FE model on link supports RMS</th>
<th>Calibrated FE model on pins &amp; rollers RMS</th>
<th>Measured RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain (µε)</td>
<td>37.9 (+19.56%)</td>
<td>32.7 (+1.15%)</td>
<td>31.3 (-1.26%)</td>
<td>31.7</td>
</tr>
<tr>
<td>Disp. (mm)</td>
<td>0.523 (+16.48%)</td>
<td>0.476 (+6.01%)</td>
<td>0.475 (+5.79%)</td>
<td>0.449</td>
</tr>
<tr>
<td>Slope (°)</td>
<td>0.038 (+15.15%)</td>
<td>0.033 (-0.00%)</td>
<td>0.033 (-0.00%)</td>
<td>0.033</td>
</tr>
<tr>
<td>Avg. RMS error</td>
<td>17.06%</td>
<td>3.05%</td>
<td>2.35%</td>
<td></td>
</tr>
</tbody>
</table>
calculated with respect to the measured frequencies. The average percentage errors show great improvements for both calibrated models. The UCF grid model supported on pins and roller has a slightly lower average percentage error compared to the model supported on link elements (springs). Overall both of the calibrated models showed great improvements in predicting responses compared to the uncalibrated model, demonstrating the robustness of the FE model updating method used.

Although, by using the pins and rollers as the boundary conditions the percentage error of the frequencies improved for $f_2$, $f_3$, $f_4$ and $f_5$, the FE model supported on the link elements had closer predicted $f_1$ with respect to the experimental frequency. Overall, the mode shapes and frequencies using the calibrated UCF grid on pins and rollers supports demonstrate a better match with the measured modal responses.

7. Discussion for full scale bridge model updating

In order to validate newly developed methodologies, it is important to have a controlled laboratory test prior to application to full scale structures. Although FE modeling of laboratory structures might be relatively straightforward compared to realistic modeling of full scale bridges, these models play an important role in better understanding of challenges in parameter estimation procedure.

There are key challenges that require careful attention prior to utilizing the proposed approach for automated FE model calibration of full scale bridges in order to ensure robust and efficient multiresponse parameter estimation. These challenges arise due to the geometric complexity, uncertain boundary conditions, unknown material properties, idealizations, loading environment, measurements, and solution of the inverse problem using multi-response data. These practical constrains for full scale bridge FE model updating are summarized in three main categories of: finite element modeling, testing and measurements, and parameter estimation.

Challenges in finite element modeling include: (1) Geometry modeling shall be based on as-built drawings, in case of availability, and field observations as accurately as possible to simulate.

**Table 10** Comparison of analytical mode shapes with experimental mode shapes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Uncalibrated FE model on link elements</th>
<th>Calibrated FE model on link elements</th>
<th>Calibrated FE model on pins and rollers</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$f_1 = 20.41 \text{ Hz}$ ($+8.11%$)</td>
<td>$f_1 = 22.23 \text{ Hz}$ ($+0.06%$)</td>
<td>$f_1 = 21.12 \text{ Hz}$ ($-4.96%$)</td>
<td>$f_1 = 22.22 \text{ Hz}$</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$f_2 = 24.44 \text{ Hz}$ ($-8.91%$)</td>
<td>$f_2 = 27.84 \text{ Hz}$ ($+3.76%$)</td>
<td>$f_2 = 26.76 \text{ Hz}$ ($-0.28%$)</td>
<td>$f_2 = 26.83 \text{ Hz}$</td>
</tr>
<tr>
<td>Mode 3</td>
<td>$f_3 = 31.32 \text{ Hz}$ ($-5.96%$)</td>
<td>$f_3 = 33.60 \text{ Hz}$ ($+0.90%$)</td>
<td>$f_3 = 33.15 \text{ Hz}$ ($-0.47%$)</td>
<td>$f_3 = 33.30 \text{ Hz}$</td>
</tr>
<tr>
<td>Mode 4</td>
<td>$f_4 = 38.07 \text{ Hz}$ ($-6.30%$)</td>
<td>$f_4 = 43.00 \text{ Hz}$ ($+5.83%$)</td>
<td>$f_4 = 42.57 \text{ Hz}$ ($+4.76%$)</td>
<td>$f_4 = 40.63 \text{ Hz}$</td>
</tr>
<tr>
<td>Mode 5</td>
<td>$f_5 = 63.20 \text{ Hz}$ ($-2.12%$)</td>
<td>$f_5 = 64.25 \text{ Hz}$ ($-0.50%$)</td>
<td>$f_5 = 64.61 \text{ Hz}$ ($+0.06%$)</td>
<td>$f_5 = 64.57 \text{ Hz}$</td>
</tr>
<tr>
<td>Avg. error</td>
<td>$6.28%$</td>
<td>$2.21%$</td>
<td>$2.11%$</td>
<td></td>
</tr>
</tbody>
</table>
realistic responses of the bridge and also ensure the best a priori FE model for initial model calibration and subsequent parameter estimations; (2) Material properties shall be verified with data available from steel manufacturers, concrete cylinder tests collected during construction, and soil properties from borehole tests. Concrete properties can also be verified with simple field measurements; (3) For the purpose of building realistic and accurate FE models, it is essential to include nonstructural elements that are present during testing and measurements of full scale bridges but normally not included in design models such as railings, asphalt layers, weight of the deck’s utility lines and reinforcement; (4) Accurate modeling of complex boundary conditions using pins, rollers, partially restrained boundary conditions, and interface elements play a major role in capturing the actual structural system behavior for full scale bridges; and (5) For more complex structures with several layers of primary and secondary elements in the superstructure and substructure, it is critical to study the desired complexity of the preliminary model with respect to the main emphasis in model updating process. The model shall be examined using different element types with coarse, medium, and fine mesh sizes in the areas of interests to fully capture the stress gradients and deformations pertinent to target areas.

Challenges in testing and measurements include: (1) Field testing is prone to higher levels of measurement noise compared to laboratory tests. Careful evaluation of acquired data sets along with filtering and removing outlier erroneous data can have a great impact in successfully executing the proposed parameter estimation procedure; (2) Environmental conditions such as temperature and humidity can alter bridge responses as well as affecting performance of data acquisition systems; and (3) Special attention is required for optimal sensor placement, data sampling rate, signal processing, and sensor technology for successful data acquisition.

Challenges in parameter estimation include: (1) Simulated parameter estimation studies are recommended prior to FE model updating in order to examine the observability and identifiability of unknown parameters in presence of modeling and measurement errors; (2) Regularization techniques play a major role in convergence of multiresponse parameter estimation procedure by applying normalization to various features at different levels. These features include unknown parameter normalization, error function normalization, objective function statistical normalization, and objective function weighting. Application of these approaches can improve robustness in finite element model calibrations; and (3) Full scale bridges have more complex geometry and structural systems which increase the size of the FE model. In order to lessen the computation time, especially with the use of the inverse problem, it is recommended to use grouping/ungrouping techniques which allow for fewer unknowns in the optimization process.

8. Conclusions and future work

In this study, a new method that uses multiresponse parameter estimation for simultaneous estimation of stiffness and mass parameters of a finite element model is presented. The FE model of a scale bridge model, UCF grid, was successfully calibrated by simultaneously using static and modal experimental data. Flexibility-based error functions were utilized for parameter estimation. Measured data, including strains, displacements, slopes, mode shapes and frequencies were used in the FE model updating. In this process two commercial software packages were used simultaneously in real time for automated and systematic optimization for finite element model updating.

Having measurement errors in experimental data is unavoidable, and the presence of these errors can affect the accuracy of the estimated parameters. Therefore, an error sensitivity analysis was performed using Monte Carlo simulations to find the parameters with the least sensitivity to measurement errors and to remove them from the parameter estimation process. In these simulations, various parameter groupings of elements with the same properties were used to reduce the number of unknowns and then ungrouping was utilized to reduce the modeling errors. In any event, changes in such parameters have minimal effects on the analytical responses of the stated problem.

A data quality study was conducted to assess the accuracy and reliability of the measured data. Based on this study, a subset of most reliable measured data was used for the parameter estimation and FE model updating of the UCF grid. Using the subset of data improved the convergence and the quality of estimated structural parameters.

Element rigidities and mass properties of the finite element model were successfully updated using the measured multiresponse data, which demonstrates the feasibility of simultaneous use of static and modal experimental data by employing various error functions. The necessity and effectiveness of different normalization techniques due to the probable ill-conditioning in the system of equations of the inverse problem were presented in this research. In this process using a set of the most observable and error tolerant unknown parameter groups, had a substantial role in this success. The initial UCF grid FE model used vertical springs to model the supports. Since these springs were identified to be in the fixed stiffness range, the model was modified to use pin and roller supports between the columns and the grid. Both calibrated models showed great improvements in predicting responses compared to the uncalibrated model. An excellent match was observed between the analytical responses and measured strain, slopes, mode shapes and frequencies. Some cases of displacement predictions showed a better match with the measured displacements compared to other cases. This can be due to measurement and/or modeling errors present in the system.

On a theoretical basis, the proposed methodology is directly applicable to full scale bridges for calibration of FE baseline models using NDT data. However, the authors acknowledge that full scale applications using more complex structures may involve more uncertainties. Thus in the future, the performance of the proposed method for full scale bridges should be studied to show that it can be used for calibrating FE models using NDT data, locating and estimating model changes as an indication of damage, and assessing retrofitting scenarios for effective rehabilitation. Since we have resolved the mathematical issues and software development related to large structures, we certainly hope to be able to apply these techniques to a full scale bridge in the future.

References

[27] DeCarlo C. Applications of statistics to minimize and quantify measurement error in finite element model updating. Master’s Thesis – Tufts University; 2008.