**Research Projects**

**Modal Identification of Nonlinear Structural Systems**

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Linear system identification methods have been successfully used by many researchers for experimental modal analysis of structures based on input-output measurements as well as operational modal analysis based on output-only measurements for the purpose of vibration-based structural health monitoring (SHM). However, these modal analyses methods are based on the assumption that measured data represent a linear dynamic response of the considered structure. Even though the modal analysis theory does not hold for nonlinear systems, it can be used as a tool to characterize specific types of nonlinear dynamic systems such as real-world civil structures with material nonlinearity based on their instantaneous/short-time modal parameters along the nonlinear response time history. These instantaneous modal parameters correspond to an equivalent linear system with stiffness equal to tangent stiffness of the nonlinear system at considered time instant. In structural and earthquake engineering communities, computation of nonlinear dynamic response of civil structures due to moderate to large amplitude excitations (common in design, response prediction, and reliability analysis of structures) is performed through linearization of nonlinear stiffness matrix at each time instant (i.e., frozen configuration). This study proposes to use a linear system identification method to estimate the instantaneous modal parameters of a nonlinear structure.

A windowed deterministic-stochastic subspace identification methods (DSI) is proposed for short-time (instantaneous) system identification of nonlinear systems when subjected to non-stationary seismic base excitations. Accuracy of this method is compared to that of the wavelet transform method when applied for identification of SDOF as well as 7-DOF systems with different material hysteretic behavior. Effects of several input factors on the accuracy of system identification results are studied. The considered input factors are: (1) type of material nonlinearity (i.e., material hysteretic behavior), (2) level of nonlinearity, (3) input excitation, and (4) length of the “short-time” data windows used in the identification. The contribution of each input factor to the total variability of two estimation error metrics is quantified through analysis-of-variance, an effect screening method.

Based on this numerical uncertainty analysis study, the following observations were made:

1. The response nonlinearity and its intensity can be tracked through identified instantaneous natural frequencies and damping ratio using DSI.  
2. The identified effective damping ratios appear to be more sensitive to structural response nonlinearity than the instantaneous natural frequencies. However, the damping ratios in general have a larger estimation uncertainty than the natural frequencies.  
3. Modal parameters obtained using DSI are consistently more accurate than those obtained using WT.  
4. The system identification results using WT are very sensitive to the input excitation. This is due to the fact that WT does not use any information about the input excitation in the identification process and therefore, its estimation error increases as the input excitation becomes more nonstationary.  
5. Estimation errors of the identified modal parameters increase for the higher modes.  
6. The type of material nonlinearity has a significant effect on the accuracy of system identification results. The identification results for nonlinear system with Giuffré-Menegotto-Pinto hysteretic models are closer to the exact values than those with bilinear hysteretic models.
(7) Use of larger data windows improves the DSI identification results of the 7-DOF systems, especially for the first vibration mode, while this input factor does not show a clear effect on identified natural frequency of the SDOF systems.

DSI is also used for short-time system identification of a full-scale seven-story shear wall structure (Figure 1) when subjected to seismic base excitation through a shake table. The structure was damaged progressively through four historical earthquake ground motions (EQ1–EQ4). Figure 2 shows the acceleration time history of input base excitations measured on the shake table for EQ2, EQ3, and EQ4, together with the hysteretic curve of base moment versus roof displacement (computed from numerical integration of measured roof acceleration) for each earthquake. Measured response data from seven longitudinal acceleration channels at the floor levels and the input acceleration measured on top of the shake table were used for short-time system identification of the test structure during three seismic tests. Figure 3(a) shows the time histories of the first mode instantaneous natural frequency of the test structure identified using 1- and 2-second windows during the three considered earthquake base excitation tests. Figure 3(b) plots the square root of an effective global stiffness estimate of the structure during the base excitation time histories. The effective global stiffness of the test structure at each time window is estimated as the secant stiffness of base moment versus roof displacement hysteretic curve during that time window (1- and 2-second). The secant stiffness is computed as the slope of straight line connecting the extreme displacement points on the hysteretic curves.

Figure 1. Full-scale shear wall test structure
Figure 2. (a) Acceleration time histories of the three considered earthquakes measured on shake table, and (b) base moment versus roof displacement of the structure during the three seismic base excitations.

Figure 3. Time histories of (a) instantaneous fundamental natural frequency, and (b) square root of an effective global stiffness measure of the test structure, estimated using 1- or 2-second data windows during the three considered earthquakes.
The following observations are made from the system identification results of this test structure.

(1) The identified instantaneous natural frequencies during the considered three earthquakes decrease drastically during the first part (with highest energy) of the strong motion and then will increase slightly as the response amplitude (i.e., level of nonlinearity) becomes smaller at the end of earthquake.

(2) The instantaneous natural frequency at the end of each earthquake is significantly smaller than that at the beginning of the earthquake. This corresponds to the stiffness degradation in the test structure during each seismic event. However, the authors would like to emphasize that the stiffness degradation is usually not correlated with the strength degradation in the structure.

(3) The identified instantaneous natural frequencies match the trend of square root effective global stiffness estimate of the structure obtained from base moment versus roof displacement hysteretic curves.

(4) The instantaneous natural frequencies at the beginning and end of each earthquake are bounded between the corresponding natural frequencies from ambient vibration and white noise tests.

(5) In general, the instantaneous natural frequencies at the beginning of each earthquake are closer to the corresponding natural frequencies identified based on the ambient vibration test data performed before the earthquake while the instantaneous natural frequencies at the end of each earthquake are closer to those identified based on white noise test data performed after the earthquake.

This study highlights the effectiveness of the DSI method for short-time (instantaneous) modal identification of nonlinear structural systems. It is expected that accurate estimates of instantaneous modal parameters to be used for characterizing the hysteretic behavior of structural components (e.g., substructures), which is the topic of an ongoing research by the research team.