Circuit Theory ES3, EE21
2017 ECE PhD Qualifier: Circuit Theory

(a) Find the transfer function for the circuit shown below, \( H(j\omega) = \frac{V_{out}}{V_{in}} \). The op amp may be considered ideal within the operating limits imposed by the DC voltage sources.

(b) Carefully and accurately sketch the Bode diagram for the magnitude of the transfer function, \(|H|\). Be sure to comprehensively label your sketch. For this Bode plot, use \( R_1=R_2=1\,\text{k}\Omega \) and \( C_1=C_2=1\,\mu\text{F} \).

(c) What is the maximum amplitude of \( V_{in} \) that this circuit may tolerate without clipping at the output? Be sure to include any assumptions or approximations made in your answer booklet.
Linear Systems Theory EE23
Honor Code: This exam represents only my own work. I did not give or receive help on this exam.

Instructions:

- Please do not turn this page until told to do so. Total time allowed for this test is 120 min.
- You should **concisely indicate your reasoning and show all relevant work** for each problem. Your score will be based on an evaluation of your understanding as reflected by what you have written for an answer.
- All work you want graded must go in the exam booklet provided. Use the extra sheets provided for scratch work only.
- There are two sheets of formulas that are attached, which you may use to solve the problem.
Problem [20 pts] In the Figure 1(a) below, we show a discrete time system consisting of a parallel combination of N Linear Time Invariant (LTI) filters with impulse response $h_k[n]$, $k = 0, 1, ..., N - 1$. For any $k$, $h_k[n]$ is related to $h_0[n]$ by the following expression,

$$h_k[n] = e^{j2\pi nk/N}h_0[n]$$

(1)

Figure 1: Figure 1(a) showing the overall system for the problem. Figure 1(b) showing the DTFT of the ideal low pass filter (in the principal interval $(-\pi, \pi]$) with cutoff frequency $\omega_c$.

Answer the following questions with clear and proper reasoning.

1. (2 pts) Is the overall system an LTI system?

2. (2 pts) State whether TRUE or FALSE - The DTFT $H_k(e^{j\omega})$ of $h_k[n]$ is periodic with period $2\pi$.

3. (4 pts) If $h_0[n]$ is an ideal discrete time low pass filter with frequency response $H_0(e^{j\omega})$ with cut-off frequency $\omega_c$ as shown in Figure 1(b), sketch the Fourier transforms of $h_1[n]$ and $h_{N-1}[n]$ for $\omega$ in the range $-\pi < \omega \leq \pi$.

4. (4 pts) Determine the value of the cutoff frequency $\omega_c$ in Figure 1(b), in terms of $N$ ($0 < \omega_c \leq \pi$) such that the system in Figure 1(a), is an identity system. That is, $y[n] = x[n]$ for all $n$ and for any input $x[n]$.

5. (4 pts) Suppose that $h[n]$ is no longer restricted to be an ideal low pass filter. If $h[n]$ denotes the impulse response of the entire system in Figure 1(a), with input $x[n]$ and output $y[n]$, then $h[n]$ can be expressed in the form

$$h[n] = r[n]h_0[n].$$

Show that (with complete and proper reasoning)

$$r[n] = N \sum_{\ell=-\infty}^{\infty} \delta[n - \ell N],$$

where $\delta[n]$ denotes the discrete time unit impulse.

6. (4 pts) From your result in the previous part, determine a necessary and sufficient condition on $h_0[n]$ such that the overall system will be an identity system. Your answer should not contain any sums.
IV. Discrete-time Fourier transform

A. Properties of the discrete-time Fourier transform

<table>
<thead>
<tr>
<th>Non-periodic signal</th>
<th>Fourier transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})e^{j\omega n} d\omega$</td>
<td>$X(e^{j\omega}) \triangleq \sum_{n=\infty}^{\infty} x[n]e^{-j\omega n}$</td>
</tr>
<tr>
<td>$a x[n] + b y[n]$</td>
<td>$a X(e^{j\omega}) + b Y(e^{j\omega})$</td>
</tr>
<tr>
<td>$x[n - n_0]$</td>
<td>$e^{-j\omega n_0} X(e^{j\omega})$</td>
</tr>
<tr>
<td>$e^{j\omega_0 n} x[n]$</td>
<td>$X(e^{j(\omega - \omega_0)})$</td>
</tr>
<tr>
<td>$x^*[n]$</td>
<td>$X^*(e^{j(-\omega)})$</td>
</tr>
<tr>
<td>$x[-n]$</td>
<td>$X(e^{j(-\omega)})$</td>
</tr>
<tr>
<td>$x_{(m)}[n] = { x[n/m], \ n \text{ multiple of } m }$</td>
<td>$X(e^{j(m\omega)})$</td>
</tr>
<tr>
<td>$x[n] * y[n]$</td>
<td>$X(e^{j\omega})Y(e^{j\omega})$</td>
</tr>
<tr>
<td>$x[n]y[n]$</td>
<td>$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)}) d\theta$</td>
</tr>
<tr>
<td>$x[n] - x[n-1]$</td>
<td>$(1 - e^{j\omega}) X(e^{j\omega})$</td>
</tr>
<tr>
<td>$\sum_{k=-\infty}^{n} x[k]$</td>
<td>$\frac{1}{1-e^{j\omega}} X(e^{j\omega}) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$</td>
</tr>
<tr>
<td>$nx[n]$</td>
<td>$j \frac{d}{d\omega} X(e^{j\omega})$</td>
</tr>
</tbody>
</table>

If $x[n]$ is real valued then

\[
\begin{align*}
X(e^{j\omega}) &= X^*(e^{j(-\omega)}) \\
\Re\{X(e^{j\omega})\} &= \Re\{X(e^{j(-\omega)})\} \\
\Im\{X(e^{j\omega})\} &= -\Im\{X(e^{j(-\omega)})\} \\
|X(e^{j\omega})| &= |X(e^{j(-\omega)})| \\
\arg\{X(e^{j\omega})\} &= -\arg\{X(e^{j(-\omega)})\}
\end{align*}
\]

Parsevals relation for non-periodic signals

\[
\sum_{n=-\infty}^{\infty} |x[n]|^2 \triangleq \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 d\omega
\]
### B. Discrete-time Fourier transform table

<table>
<thead>
<tr>
<th>$x[n]$</th>
<th>$X(e^{j\omega})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta[n]$</td>
<td>1</td>
</tr>
<tr>
<td>$\delta[n-n_0]$</td>
<td>$e^{-j\omega n_0}$</td>
</tr>
<tr>
<td>$\sum_0^\infty \delta(n-kN)$</td>
<td>$\frac{2\pi}{N} \sum_0^\infty \delta \left( \omega - \frac{2\pi k}{N} \right)$</td>
</tr>
<tr>
<td>1</td>
<td>$2\pi \sum_0^\infty \delta(\omega - 2\pi k)$</td>
</tr>
<tr>
<td>$e^{j\omega_0 n}$</td>
<td>$2\pi \sum_0^\infty \delta(\omega - \omega_0 - 2\pi k)$</td>
</tr>
<tr>
<td>$\cos \omega_0 n$</td>
<td>$\pi \sum_0^\infty [\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)]$</td>
</tr>
<tr>
<td>$\sin \omega_0 n$</td>
<td>$\frac{\pi}{j} \sum_0^\infty [\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)]$</td>
</tr>
<tr>
<td>$u[n]$</td>
<td>$\frac{1}{1 - e^{-j\omega}} + \pi \sum_0^\infty \delta(\omega - 2\pi k)$</td>
</tr>
<tr>
<td>$a^n u(n)$, $</td>
<td>a</td>
</tr>
<tr>
<td>$(n+1) a^n u[n]$, $</td>
<td>a</td>
</tr>
<tr>
<td>$\frac{(n+m-1)!}{n!(m-1)!} a^n u[n]$, $</td>
<td>a</td>
</tr>
<tr>
<td>$\frac{1}{1 - a^2} a^n u[n]$, $</td>
<td>a</td>
</tr>
<tr>
<td>$\left{ \begin{array}{ll} 1, &amp;</td>
<td>n</td>
</tr>
<tr>
<td>$\left{ \begin{array}{ll} 1, &amp;</td>
<td>n</td>
</tr>
<tr>
<td>$\left{ \begin{array}{ll} 1, &amp;</td>
<td>\omega</td>
</tr>
</tbody>
</table>
Digital Electronics ES4, EE14, EE26
Digital Logic

1) BCD is binary coded decimal. It encodes each of the decimal digits in 4 bits. Please design a binary to BCD decoder that can decode the numbers 0 through 15 in binary into BCD. Your design should show the truth table and utilize “don't care “conditions if necessary. Show all the kmaps and final equations. Show a schematic of your final design.

2) Design the most minimized form of a mod-4 counter that counts through the sequence 5,3,1,0 and repeats. All unused states should go to state 0. Use only JK positive edge triggered flip flops. Show all your kmaps and final equations. Draw your final Schematic

3) A 16K memory mapped system has 2k of RAM followed by another 4k of SRAM, followed by another 8k of ROM. Show the memory map and the starting/ending locations of each memory partition, including any unused memory in both binary and in hexadecimal.
Analog Electronics EE21, EE22
Problem 1 (20 points): Consider the circuit in Figure 1 with transistors operating in saturation and threshold voltage, dimensions and transconductance parameter given. Find the value of $V_1$, $V_2$, and $V_3$.

Consider the voltage amplifier circuit with feedback in Figure 2. The NMOS transistor has transconductance $g_m$, $\lambda = 0$, and the source-to-bulk voltage $V_{sb} = 0$.

Problem 2 (20 points): Sketch the (low frequency) small-signal equivalent circuit.

Problem 3 (40 points): For the circuit in Figure 2, assume $(R_1 + R_2) \gg R_D$. Using (low frequency) small signal analysis, find an expression for the following:

a. Open-loop gain: $A = V_O/V_I$

b. Feedback factor: $\beta = V_F/V_O$

c. Closed-loop gain: $A_F = V_O/V_S$

d. Simply the expression for the closed-loop gain $A_F$ under the assumption that $A\beta \gg 1$.

Problem 4 (20 points): Write an expression for the frequency of the pole at the source node of the transistor in Figure 2.
Figure 1. Circuit for Problem 1.

Figure 2. Circuit for Problems 2, 3, and 4.
Electromagnetic Fields and Waves EE18
Problem 1: A coaxial cable consists of two concentric long hollow cylinders of zero resistance; the inner has radius $a$, the outer has radius $b$, and the length of both is $l$, with $l \gg b$, as shown in the figure. The cable transmits DC power from a battery to a load. The battery provides an electromotive force $\varepsilon$ between the two conductors at one end of the cable, and the load is a resistance $R$ connected between the two conductors at the other end of the cable. A current $I$ flows down the inner conductor and back up the outer one. The battery charges the inner conductor to a charge $-Q$ and the outer conductor to a charge $+Q$.

(a) Find the direction and magnitude of the electric field $\mathbf{E}$ everywhere.
(b) Find the direction and magnitude of the magnetic field $\mathbf{B}$ everywhere.
(c) Calculate the Poynting vector $\mathbf{S}$ in the cable.
(d) By integrating $\mathbf{S}$ over appropriate surface, find the power that flows into the coaxial cable.
(e) How does your result in (d) compare to the power dissipated in the resistor?

Problem 2: If you have a 10m long lossless transmission line with a characteristic impedance of 100 ohms connected to a 1kW generator with an impedance of $200 + i100$ ohms that puts out a 1GHz signal and a load of $50 + i250$ ohms, answer the following questions.

a) How much power is transmitted to the load?

b) How much power is reflected back and transmitted into the generator?

c) By adding two shorted stubs to the transmission line remove all reflections in this system. Show all the work on the attached smith charts and all calculations performed. Note the length of the stub and its position for both stubs.
Communications systems EE107
Problem 1 Caches (15 pts)

A test application accesses the following memory addresses (8-bit addresses), shown in hex:

0x02, 0x06, 0x0E, 0x03, 0x12, 0x06, 0x03, 0x0E

The L1 cache has the following parameters:

- Word size one byte
- Block size 2 bytes
- Cache capacity: 12 bytes
- Associativity: 3-way set associative
- Physical address size: 8-bits
- Cache Eviction Policy: LRU

a.) Calculate the size of the Tag, index and offset fields in bits. Draw a table that translates each address into binary digits and shows the separate tag, index, and offset fields. (3pts)

b) Record each access as a hit or miss in the table above. Draw a diagram showing the final contents of the data and tag portion of the cache below. The cache is initially empty. (4pts)

c) The pattern above has a significant number of misses. Why and what type of misses? (2pts)

d) How many bits would you need for the tag, index and offset if you switched to a direct-mapped cache with the same block size and cache size (in words) as the first cache configuration. Show the new cache block diagram, showing among other things, which index bits decode to select each set. (2pts)

f) Propose a new cache configuration that improves the hit rate. (2pts)

g) Describe possible costs (performance, power, area) of your new cache configuration. (2pts)
EE 126 Qualification Exam Question, January 2017 (page 2)
Problem 2 Pipelining. (10 points)

The code segment will execute on a 32-bit, 5-stage MIPS processor with the following features:

- The processor includes the following stages instruction fetch (IF), instruction decode (ID), execute (EX), memory access (MEM), and write back (WB).
- The processor has full forwarding and bypassing logic can forward data from the EX/MEM and MEM/WB pipeline registers directly to the inputs to the ALU or MEM stage.
- The processor supports simple static branch prediction by always predicting not taken.
- Branches are resolved in the EX stage, speculative state is flushed.

All registers are loaded before the start of the code segment and instructions issue in program order.

LW  $t1, 0($s0)
ADDI  $t1, $t1, x108A
LW  $t2, 4($s0)
SUB  $t1, $t1, $t2
ADDI  $t1, $t2, x1010
ADD  $t1, $t2, $t1

a) How long, in cycles, will it take to complete the code segment on the MIPS processor above? Hint: draw a pipeline diagram. (3 pts)

b) The forwarding logic will be used pretty extensively. In your pipeline diagram draw an arrow to indicate each instance of forwarding. List each instance of forwarding in a table like the one below, circle the register. Note: the example below is not found in the code. (2pts)

<table>
<thead>
<tr>
<th>Source Instruction and RegisterID</th>
<th>Destination Instruction and RegisterID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td></td>
</tr>
<tr>
<td>SUB, $t0, $t1, $t2</td>
<td>ADD, $t4, $t0, $t5</td>
</tr>
</tbody>
</table>

c) While executing the code segment, the pipeline had to stall. Reorder the code segment to remove stalls on your 5-stage pipeline. (2pt)

d) Besides the data-dependencies you indicated above the code segment includes anti-dependencies and output-dependencies. List all of these non-data dependencies and their type found in the original code segment using a similar table to that of part b. (1pt)

e) Do these non-data dependencies (anti-dependencies, output-dependencies) impact the performance of the 5-stage pipeline, why or why not? In what type of processor architecture will these dependencies cause hazards? Name a technique used to remove these hazards. (2pt)
Operating Systems EE128
1. A part of the operating system is "resident" if it is always in memory, and "virtual" if it can be stored instead in virtual memory on disk. Which parts of an operating system must be resident, and which can be virtual? Why?

2. The Linux disk journal is an apparent contradiction of the basic principle that the disk driver manages all device state. What states of the disk are controlled by the actual driver, and which are controlled by the journalling subsystem in an application program? Why?
Programming COMP11, COMP15
Probability and Statistics

Here we consider a random variable $G$ whose probability density function (PDF) depends on a parameter, call it $H$. We start by modeling this problem under the assumption that $H$ is random with a given PDF. In this case, it makes sense to think of the PDF for $G$ as being conditioned on $H = h$. To be concrete, consider the case where the two PDFs are

$$f_G|H(g|h) = \begin{cases} \frac{1}{2} & 0 \leq g \leq h \\ a & 0 \leq g \leq h \\ 0 & \text{else} \end{cases}$$

$$f_H(h) = \begin{cases} \frac{1}{4} & 0 \leq h \leq b \\ \frac{7}{4} & b \leq h \leq c \\ 0 & \text{else} \end{cases}$$

with constants $a$, $b$, and $c$.

1. Determine the constant $a$.
2. Show that $b = 1/2$, and $c = 1$ if we know that $E[H] = 11/16$.
3. Determine the joint distribution for $G$ and $H$.
4. Determine the marginal distribution for $G$.

Now we consider the problem where somehow we know $H$ is either $1/4$ or $3/4$ and must make a choice as to which based on data.

5. Given a value for $G$, what is the maximum likelihood rule for determining whether $H$ is $1/4$ or $3/4$?
6. Suppose now that we are given a series of independent and identically distributed samples $G_i$ for $i = 1, 2, ..., N$. What is the maximum likelihood rule for choosing between the two values for $H$ based on the number of times $G_i > .5$ from the $N$ samples? Simplify your results as much as possible.
Semiconductors EE-113, EE-114
Problem 1: Explain the difference between extrinsic and intrinsic semiconductors.

Problem 2: The number of electron-hole pairs in intrinsic germanium (Ge) is given by: $n_i = 9.7 \times 10^{15} \frac{T^{3/2}}{e^{E_g / 2kT}} \text{[cm}^3\text{]}$ ($E_g = 0.72 \text{ eV}$)

a) What is the density of pairs at $T = 20^\circ\text{C}$?

b) Will undoped Ge be a good conductor at $200^\circ\text{C}$? If so, why?

Problem 3: If no electron-hole pairs were produced in germanium (Ge) until the temperature reached the value corresponding to the energy gap, at what temperature would Ge become conductive? ($E_{th} = 3/2 kT$)

Problem 4: The energy gap $E_g$ of ZnSe is $2.3 \text{ eV}$.

a) Is this material transparent to visible radiation? Substantiate your answer.

b) How could you increase the electrical conductivity of this material? Give the reasons for the effectiveness of your suggested approach.
Consider the following discrete-time, Linear Time Varying system written in state-space:

$$x_{k+1} = P_k x_k + B u_k, \quad k \geq 0, \quad (1)$$

where the time-varying system matrix, $P_k$, is such that

$$P_k = \begin{cases} P_1, & k \text{ is odd}, \\ P_2, & k \text{ is even}, \end{cases} \quad (2)$$

with

$$P_1 = \frac{1}{20} \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}, \quad P_2 = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad x_0 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}. \quad (3)$$

Let $x_\infty$ be defined as

$$x_\infty \triangleq \lim_{k \to \infty} x_k.$$

**Answer the following questions:**

1. Find $x_\infty$, when there is no input, i.e., $u_k = 0, \forall k$.
2. Find $x_\infty$, when the input is a step function, i.e.,

$$u_k = \begin{cases} 1, & k \geq 0, \\ 0, & \text{otw.} \end{cases}$$

**Hint 1:** A discrete-time Linear Time Invariant system is such that

$$x_{k+1} = P x_k + B u_k, \quad (4)$$

where $x_\infty$ for a step input is given by

$$x_\infty = (I - P)^{-1} B, \quad (5)$$

provided that $P$ is stable. Do not use Eq. (5) without justifying it, you may assume the following:

$$\sum_{k=0}^{\infty} P^k = (I - P)^{-1}. \quad (6)$$

**Hint 2:** A $2 \times 2$ matrix inverse is given by:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \quad (7)$$