STRENGTH

\[ F \cdot \Delta = \text{WORK} \]
(F AND \( \Delta \) ARE VECTORS)

STABILITY

\[ P \cdot \Delta = 0 \] (WORK)
(P AND \( \Delta \) ARE VECTORS)

STRUCTURAL DESIGN IS THE ART AND SCIENCE OF CREATING STRUCTURAL SYSTEMS WITH ADEQUATE STRENGTH, STIFFNESS, STABILITY AND DUCTILITY.

OUR RESPONSIBILITY IS TO DESIGN STRUCTURES THAT ARE USEFUL AND SAFE.

OUR CALLING IS TO DESIGN STRUCTURES THAT ARE EFFICIENT, ECONOMICAL AND ELEGANT.
Moment of Inertia

Moment about ENA

\[ \sum \sigma_y (dA) \cdot y = \text{moment arm} \]

\[ \sigma_y (dA) = \frac{\sigma}{c} y (dA) \]

\[ M = \int \frac{\sigma}{c} y^2 dA \]

Define \( I \) such that

\[ \sigma_y = \frac{M y}{I} \]

\[ I = \int y^2 dA \]

= Geometric

flexural stiffness

parameter for a given section

Define \( S \) such that

\[ \sigma_{max} = \frac{M}{S} \]

\[ S = \frac{I}{d-c} \]

= Flexural strength

parameter
Galileo begins thinking about the strength of his cantilever by imagining how he might lift a heavy stone in his garden.

Galileo's stone is a bit easier to grasp mathematically if it is represented as a free-body diagram.

\[
\begin{align*}
G \sum F_B &= 0 = -P_A (BA) + V_c (BC) \\
G \sum M_N &= 0 = V_c (CN) - P_g (NG)
\end{align*}
\]

\[
P_g = P_a \frac{(BA)(CN)}{(BC)(NG)}
\]

Next, Galileo imagines the action of a cantilever prying out of a wall as similar to the action of the stone prying out of the ground.
Galileo set up his bending problem as an overturning problem. A stone column has uniform stresses at its base and rotates about its edge.

Galileo assumed point B to be the point of rotation, just as he had done for the stone in his garden. He further assumed, based on the stone in his garden, that the bending stresses were uniformly distributed.

Internal resisting moment:

\[ M = \frac{f_{\text{max}}}{h^2} \cdot (b \cdot h) = \frac{f_{\text{max}}}{h^2} \cdot \frac{bh^2}{2} \]

When \( F = \frac{f_{\text{max}}}{h^2} \cdot \frac{bh^2}{2} \)
GALILEO (1638)

\[ M = \sigma_{\text{max}} bh \cdot \frac{h}{2} = \sigma_{\text{max}} b h^2 / 2 \]

COULOMB (1773)

\[ M = \sigma_{\text{max}} bh \cdot \frac{2h}{3} = \sigma_{\text{max}} b h^2 / 6 \]

\[ \sigma_y = \frac{\sigma_{\text{max}}}{h/2} \]

\[ \sigma_{\text{max}} = \frac{\sigma_y h/2}{y} = \frac{M}{s} \]

\[ \sigma_y = \frac{M y}{s \cdot h^2 / 3} = \frac{My}{t} \]

For a rectangular section: \[ I = s h^2 / 12 \]
LOADS

DEAD
- CONC + DECK = 48 psf
- CARPET = 2 psf
- PONDING = 5 psf
- FRAMING = 5 psf
- HUNG PARTITIONS = 20 psf
- OFFICE = 50 psf

LINE

STEEL PROPERTIES
- Fy = 50 ksi
- Fu = 29,000 ksi

TYPICAL DECK-SLAB
- 1 1/2" = 1' 0"

BEAM DESIGN - STEEL LENGTH

\[ q = (10 \text{ psf} + 50 \text{ psf}) \times 8' = 112 \text{ kips} \]

\[ M = \left( \frac{112 \times 20}{8} \right) = 568 \text{ ft-kips} \]

\[ V = \frac{112}{2} \times 20 = 112 \text{ kips} \]

\[ S \geq \frac{568 \times 12'}{(0.66)(50 \text{ kips})} = 20.4' \text{ min} \]

Note: W12x19 has \( S_{y} \) = 21.3"
CHECK BEAM STIFFNESS

WEIGHT - SLAB IS PLACED LEVEL

\[ D_W = 48 \text{ psf} + 5 \text{ psf} + 5 \text{ psf} = 58 \text{ psf} \]
\[ L_W = 20 \text{ psf} \quad \text{(STRENGTH ONLY, USE FOR COMPOSITE BEAMS)} \]
\[ q_W = \frac{(58 \text{ psf} \times 8')}{1000' \text{ k}} = 0.464' \]
\[ M_W = \frac{(0.464' \times (20')^2}{8} = 23.2' \]
\[ \Delta_W = \frac{5(0.464') (20)' (728' \frac{3}{8})}{384 (29000' \text{ k})(130'')} = 0.44'' \leq 0.542 < 0.660'\]

SUPERIMPOSED DEAD + LIVE

\[ D_L = 2 \text{ psf} + 10 \text{ psf} + 20 \text{ psf} = 32 \text{ psf} \]
\[ L = 50 \text{ psf} \]
\[ q_L = \frac{(32 \text{ psf} + 50 \text{ psf}) \times 8' }{1000' \text{ k}} = 0.656' \]
\[ M_L = \frac{(0.656' \times (20')^2}{8} = 31.3' \]
\[ \Delta_L = \frac{5(31.3') (20)' (728' \frac{3}{8})}{48 (29000' \text{ k})(130'')} = 0.60'' \leq 0.400 < 0.560'\]

GIRDERS DESIGN - STRENGTH

\[ \begin{align*}
&8' \quad 8' \quad 8' \quad 8' \quad 8' \\
&24k \quad 24k \quad 24k \quad 24k \\
&48k \quad 48k \quad 576k \quad 384k \\
\end{align*} \]

\[ 48h(8') = 384k' \]
\[ 48h(16') = 24k(8') = 576k' \]

\[ S \geq \frac{576k'(12'')}{33.6's;} \quad \text{has} \quad S = 213'^3 \]
CHECK GIRDER STIFFNESS

\[ \Delta_t = \frac{5 (576)(40)^2(1728)}{48 (29000)(2850)} = 2.01" = \frac{1}{240} < \frac{1}{80} \text{ ok} \]

TRY LINE LOAD REDUCTION \( A_t = 32'(20') = 640' \)

\[ L = 50 \text{psf} \left( 0.25 + \frac{15}{\sqrt{2} (640^{1/2})} \right) = 33.5 \text{psf} \quad \text{(ASCE 7.05)} \]

\[ q = \frac{(70 \text{psf} + 34 \text{psf})(8)}{1000 \text{psf}} = 0.992 \text{ kips/ft} \]

\[ 0.992 \text{ kips/ft} (20') = 19.8 \text{k}\]

\[ S = \frac{19.8 (209^3)}{24} = 172"^3 \quad \text{(124 x 76 HAS} \quad S_4 = 176"^3) \]

\[ I = \frac{(19.8) 5 (576)(40)^2(1728)}{24 (48 (29000)(2.67)}) = 1768"^4 \]

\[ \Delta_s = \frac{(32 + 34) (19.8)}{90 + 34} \frac{5 (576)(40)^2(1728)}{48 (29000)(2100)} = \frac{1.20"}{0.437} = \frac{1.20"}{0.437} \]

\[ \text{COMPARE TO MORE PRECISE} \Delta_s \]

\[ \Delta_s = \frac{1728}{(29000)(2100)} \left( \frac{(2536)(y)(18') + (1694)(8')(12')}{2} + (848)(8')(13.3) \right) = 0.517" + 0.587" + 0.102" = 1.21" \]

\[ 1.20" - 1.21" = 0.008 = -0.8\% \]
BEAM

\[ q = \left( \frac{90 + 50}{10} \right) \frac{12}{1000} = 1.4 \] 
\[ M = 1.4 (40)^2 A = 280 k \cdot ft \]
\[ S = 280 (12) = \frac{102}{3} \] 
\[ S_c = \frac{115}{3} \] 
\[ I_e = 1360 \] 
\[ \Delta_c = \frac{8200}{40^2} \cdot 5 (280) (40) \left( \frac{1728}{14000} \right) \]
\[ = \frac{120}{40} \leq \frac{7860}{40} \]

GIRDERS

\[ S \geq 280 (12) = \frac{102}{3} \] 
\[ \Delta_c \leq \frac{B}{E} \text{ INSPECTION} \]

HT OF 40'0" X 20'0" BEAMS

\[ P_1 = 12 (20 \times 7.2) = 8450 \] 
\[ + 2 (40) (76) = 6080 \] 
\[ 10640 \] 
\[ (50') (40') = 5.32 \text{ psi} \]

\[ P_4 = 6 (40) (55.5) = 13200 \] 
\[ + 2 (20) (55.5) = 2200 \] 
\[ 15400 \] 
\[ (80') (30') = 6.42 \text{ psi} \]
**Subject:** CEE-24  **PLATE GIRDERS**

\[ S = 12,000 \left( \frac{12}{5} \right) = 4360 \text{ in}^3 \]

W36 x 200 has \( S = 3040 \text{ in}^3 \) \( N4 \)

\[ S = A_f d \geq 4360 \text{ in}^3 \]

\[ \frac{L}{(2A_f + d) / 490}{(1/44)} \]

**MINIMIZE \( L \) SUBJECT TO ABOVE CONSTRAINTS**

<table>
<thead>
<tr>
<th>( d )</th>
<th>( A_f )</th>
<th>( K_0 )</th>
<th>( FLG )</th>
<th>( WEB )</th>
<th>( W_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>36&quot;</td>
<td>121.2&quot;</td>
<td>947.5&quot;</td>
<td>46&quot;x2 3/8&quot;</td>
<td>3 3/8&quot;</td>
<td>887.5&quot;</td>
</tr>
<tr>
<td>48&quot;</td>
<td>70.8&quot;</td>
<td>782.5&quot;</td>
<td>40&quot;x2 3/4&quot;</td>
<td>3 3/8&quot;</td>
<td>742.5&quot;</td>
</tr>
<tr>
<td>60&quot;</td>
<td>72.7&quot;</td>
<td>697.5&quot;</td>
<td>36&quot;x2&quot;</td>
<td>3 3/8&quot;</td>
<td>704.5&quot;</td>
</tr>
<tr>
<td>72&quot;</td>
<td>60.6&quot;</td>
<td>657.5&quot;</td>
<td>30&quot;x2&quot;</td>
<td>3 3/8&quot;</td>
<td>726.5&quot;</td>
</tr>
<tr>
<td>84&quot;</td>
<td>51.9&quot;</td>
<td>639.5&quot;</td>
<td>24&quot;x2 3/4&quot;</td>
<td>3 3/8&quot;</td>
<td>808.5&quot;</td>
</tr>
<tr>
<td>96&quot;</td>
<td>45.4&quot;</td>
<td>636.5&quot;</td>
<td>24&quot;x2&quot;</td>
<td>1 3/4&quot;</td>
<td>874.5&quot;</td>
</tr>
<tr>
<td>108&quot;</td>
<td>40.4&quot;</td>
<td>642.5&quot;</td>
<td>20&quot;x2&quot;</td>
<td>2&quot;</td>
<td>980.5&quot;</td>
</tr>
<tr>
<td>120&quot;</td>
<td>36.3&quot;</td>
<td>656.5&quot;</td>
<td>24&quot;x1 1/2&quot;</td>
<td>2 3/4&quot;</td>
<td>1140.5&quot;</td>
</tr>
</tbody>
</table>

**ACTUAL DESIGN WILL BE SLIGHTLY STRONGER**

**MAY HAVE A THINNER WEB**

\[ t_f \leq 2 \times 0.38 \sqrt{\frac{29000}{50 \text{ psi}}} = 18.3 t \]  

(TAB. B4.1, P. 161-16)

\[ \frac{t}{\sqrt{13}} \]  

**ACTUALLY APPLIED TO BOLTED I-SHAPES ONLY**

\[ t_{w2} \geq \sqrt{2.24 \sqrt{\frac{29000}{50}}} = \sqrt{53.9} \]  

(G2.1, G2.2, P. 161-65)

---

30 by 38 blocks at .25 inches
DEEPLY HEELS BUCKLE AT LOWER SHEAR STRESSES, BUT HIGHER SHEAR AREA

(i) \( \frac{h}{x_w} \leq 1.10 \sqrt{\frac{5(29000)}{50}} = 59.2 \)  \( (G2-3, p.16.1-65) \)

\( x_w \geq h, \ C_v = 1.0 \)

\( 59.2 \)

(ii) \( \frac{1.10 \sqrt{5(29000)}}{50} = 59.2 < \frac{h}{x_w} \leq \frac{73.8}{59.2} = 1.37 \sqrt{\frac{5(29000)}{50}} \)

\( h \leq x_w \leq \frac{h}{73.8}, \ C_v = \frac{1.10 \sqrt{5(29000)}}{50} \frac{h}{x_w} \)

\( = 59.2 \left( \frac{x_w}{h} \right) \)  \( (G2-4, p.16.1-65) \)

(iii) \( \frac{h}{x_w} > 73.8, \ C_v = \frac{1.81(29000)(5)}{(4x_w)^2(50)} \)

\( x_w < \frac{h}{73.8}, \ C_v = 4380 \left( \frac{x_w}{h} \right)^2 \)  \( (G2-5, p.16.1-65) \)

\[ V_a = \frac{0.6(606)}{15} \cdot A_{0v} \frac{C_v}{2065} \frac{A_{0v}}{2065} \]

**For**  \( d = 120'' \)

\( \text{TRY} \ x_w = 1'' \)

\( h = 117'' = 1'' \)

\( x_w = 1'' \)

\( C_v = 4380 \left( \frac{1''}{117''} \right)^2 = 0.320 \)

\( V_a = 2065 \cdot (120 - 2) \cdot (0.320) = 7686 > 4006 \)  \( \Box \)
\[ t_{min} \geq \frac{1}{2}'' \]

**NO WEB STIFFENERS**  \( k_0 = 5.0 \)

**ITERATION #2**

<table>
<thead>
<tr>
<th>( d )</th>
<th>( d_w2 )</th>
<th>( A_v2 )</th>
<th>( V_{s2} )</th>
<th>( C_{u2} )</th>
<th>( V_{o2} )</th>
<th>( D/C_2 )</th>
<th>( H_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>36''</td>
<td>3''</td>
<td>18''²</td>
<td>61.5</td>
<td>0.963</td>
<td>347k</td>
<td>1.15</td>
<td>874k</td>
</tr>
<tr>
<td>48''</td>
<td>3''</td>
<td>24''²</td>
<td>87</td>
<td>0.579</td>
<td>278k</td>
<td>1.44</td>
<td>667k</td>
</tr>
<tr>
<td>60''</td>
<td>3''</td>
<td>30''²</td>
<td>112</td>
<td>0.399</td>
<td>210k</td>
<td>1.71</td>
<td>585k</td>
</tr>
<tr>
<td>72''</td>
<td>3''</td>
<td>36''²</td>
<td>107</td>
<td>0.370</td>
<td>333k</td>
<td>1.20</td>
<td>553k</td>
</tr>
<tr>
<td>84''</td>
<td>3''</td>
<td>42.5''²</td>
<td>127</td>
<td>0.291</td>
<td>284k</td>
<td>1.41</td>
<td>537k</td>
</tr>
<tr>
<td>96''</td>
<td>3''</td>
<td>48''²</td>
<td>123</td>
<td>0.291</td>
<td>417k</td>
<td>0.95</td>
<td>561k</td>
</tr>
<tr>
<td>108''</td>
<td>1''</td>
<td>81''²</td>
<td>187</td>
<td>0.288</td>
<td>361k</td>
<td>1.08</td>
<td>538k</td>
</tr>
<tr>
<td>120''</td>
<td>1''</td>
<td>120''²</td>
<td>117</td>
<td>0.320</td>
<td>763k</td>
<td>0.52</td>
<td>643k</td>
</tr>
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</table>

**ITERATION #3**

<table>
<thead>
<tr>
<th>( d )</th>
<th>( d_w3 )</th>
<th>( A_v3 )</th>
<th>( V_{s3} )</th>
<th>( C_{u3} )</th>
<th>( V_{o3} )</th>
<th>( D/C_3 )</th>
<th>( H_3 )</th>
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<tbody>
<tr>
<td>36''</td>
<td>3''</td>
<td>27''²</td>
<td>41.0</td>
<td>1.00</td>
<td>540k</td>
<td>0.74</td>
<td>704k</td>
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<tr>
<td>48''</td>
<td>3''</td>
<td>30''²</td>
<td>67.6</td>
<td>0.851</td>
<td>510k</td>
<td>0.78</td>
<td>705k</td>
</tr>
<tr>
<td>60''</td>
<td>3''</td>
<td>45''²</td>
<td>74.7</td>
<td>0.786</td>
<td>707k</td>
<td>0.57</td>
<td>683k</td>
</tr>
<tr>
<td>72''</td>
<td>3''</td>
<td>54''²</td>
<td>90.7</td>
<td>0.533</td>
<td>575k</td>
<td>0.79</td>
<td>532k</td>
</tr>
<tr>
<td>84''</td>
<td>3''</td>
<td>63''²</td>
<td>106</td>
<td>0.390</td>
<td>491k</td>
<td>0.81</td>
<td>570k</td>
</tr>
<tr>
<td>96''</td>
<td>3''</td>
<td>72''²</td>
<td>123</td>
<td>0.291</td>
<td>417k</td>
<td>0.95</td>
<td>561k</td>
</tr>
<tr>
<td>108''</td>
<td>2''</td>
<td>94.5''²</td>
<td>119</td>
<td>0.310</td>
<td>586k</td>
<td>0.68</td>
<td>592k</td>
</tr>
<tr>
<td>120''</td>
<td>2''</td>
<td>105.5''²</td>
<td>134</td>
<td>0.245</td>
<td>514k</td>
<td>0.78</td>
<td>593k</td>
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**SECTION PROPERTIES**

<table>
<thead>
<tr>
<th>( d )</th>
<th>( I_o )</th>
<th>( S_o )</th>
<th>( I_s )</th>
<th>( S_s )</th>
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</thead>
<tbody>
<tr>
<td>36''</td>
<td>69860''⁴</td>
<td>3360''³</td>
<td>69260''⁴</td>
<td>3350''³</td>
</tr>
<tr>
<td>48''</td>
<td>101000''⁴</td>
<td>4210²</td>
<td>98600''⁴</td>
<td>4110²</td>
</tr>
<tr>
<td>60''</td>
<td>136000''⁴</td>
<td>4810²</td>
<td>132000''⁴</td>
<td>4400²</td>
</tr>
<tr>
<td>72''</td>
<td>173000''⁴</td>
<td>5290²</td>
<td>167000''⁴</td>
<td>4630²</td>
</tr>
<tr>
<td>84''</td>
<td>222000''⁴</td>
<td>5290²</td>
<td>212000''⁴</td>
<td>5050²</td>
</tr>
<tr>
<td>96''</td>
<td>277000''⁴</td>
<td>3770²</td>
<td>261000''⁴</td>
<td>5430²</td>
</tr>
<tr>
<td>108''</td>
<td>318000''⁴</td>
<td>3770²</td>
<td>314000''⁴</td>
<td>5670²</td>
</tr>
<tr>
<td>120''</td>
<td>358000''⁴</td>
<td>6440²</td>
<td>370000''⁴</td>
<td>6160²</td>
</tr>
</tbody>
</table>

30 by 38 blocks at .25 inches
CHECK STIFFNESS REQUIREMENTS

\[ I \geq \frac{800(60^2)(1722)}{48(290006)(2)} = 107,000''^4 \]

\[ \frac{L}{360} = 2'' \]

ASSUME SUPERIMPOSED LOADS AND ABSOLUTE DEFLECTION LIMITS CANCELED OUT

BEADJUST PARAMETERS TO SATISFY STRENGTH AND STIFFNESS REQ.

<table>
<thead>
<tr>
<th>d</th>
<th>h4</th>
<th>k'k4</th>
<th>k'w</th>
<th>h'w</th>
<th>Awy</th>
<th>Gwy</th>
<th>Vwy</th>
<th>Wwy</th>
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<tr>
<td>36</td>
<td>46</td>
<td>3.2</td>
<td>58</td>
<td>10,900</td>
<td>6,880</td>
<td>22.5</td>
<td>1.00</td>
<td>450</td>
</tr>
<tr>
<td>48</td>
<td>46</td>
<td>2.6</td>
<td>58</td>
<td>10,800</td>
<td>4,990</td>
<td>30</td>
<td>0.380</td>
<td>316</td>
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<tr>
<td>60</td>
<td>30</td>
<td>2.2</td>
<td>58</td>
<td>13,800</td>
<td>4,820</td>
<td>47.5</td>
<td>0.673</td>
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<td>2</td>
<td>34</td>
<td>16,700</td>
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<td>34</td>
<td>19,500</td>
<td>4,610</td>
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<td>38</td>
<td>25,000</td>
<td>4,710</td>
<td>94.5</td>
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<td>576</td>
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<td>1</td>
<td>38</td>
<td>26,000</td>
<td>4,360</td>
<td>105</td>
<td>0.241</td>
<td>506</td>
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\[ W \]

STIFFNESS CONTROLS

STRENGTH CONTROLS

50 by 50 blocks at .25 inches
MAKING TRG 72 x 581 COMPOSITE

Leff = 60" / 4 = 15.0" = 180"

4.5" NWC S1458
4000 psi
E0 = 36000 ksi
n = 22000 = 8.66 = 8
3600

STRENGTH CONTROLS
50 ksi (171") = 8550 kips
3850 kips = 3.10

I2 = 3" x 2.25" = 7.25"

M0 = 2750 kips (5.25" x 1.93") = 19700 kips
+ 50 (30 x 1.93) / 2
+ 50 (30 x 0.07) / 2
+ 50 (0.75) (68") (34.1")
+ 50 (30) (1/2) (67.1")
+ 50 (30) (1/2) (67.1")
+ 31600 kips = 26400 kips

26400 kips (0.9) = 15800 kips

1.5

4630 sq. (36,000) = 12700 kips

12"

(15800 - 12700) / 12700 = 24% STRG INCREASE

2 (2750 / 11.5) = 478 STUBS REPO
A structure is unstable if, when its load or shape is increased slightly, it deforms substantially.

A column under axial load is the classic stability problem.

\[ \sum M_a = 0 \neq Pd \text{ no resistance} \]

This column is unstable \( Pd \) as soon as you squeeze on it, it will begin to tip and the hinge at its base will not resist the overturning force \( M = Pd \). It is like trying to balance your pencil on its end.

Add a string to the top of the column.

Apply a perpendicular load \( H \) to displace the column a distance \( \Delta \).

\[ F = K \Delta = KA \]

\[ \sum M_o = 0 = -Pd + FL \rightarrow Pd = FL = K\Delta L \]

\[ P_{cr} = KL \]
\[ P_{cr} = \text{Critical Buckling Load. This is the axial load which, if applied to the column, will maintain the displacement (perturbation) } \Delta. \]

For \( P < P_{cr} \) 

The column will snap back to its upright position after removing the horizontal load required to displace it a distance \( \Delta \).

\[ P = P_{cr} \]

This is called the bifurcation load. Under this load, the column will maintain its displaced shape without the help of any applied horizontal load, \( h \).

For \( P > P_{cr} \) 

This axial load will displace the column even further, into its non-linear geometric range, after the perturbation load \( h \) is removed.

The bending stiffness of a point-loaded cantilever:

\[ K = \frac{3EI}{L^3} \]

\[ P_{cr} = KL = \frac{3EI}{L^3} \]

Actually, \( P_{cr} = \frac{\pi^2EI}{4L^2} = \frac{2.47EI}{L^2} \)

\( P_d \) acts on the column over all displacement \( \Delta \) after the column has been displaced.

\( P_d \) acts on the curved member.

\[ \Delta = \frac{PL^3}{3EI} \]

\[ F = K \Delta \]

\[ K = \frac{H}{\left(4H/EI+5EIL^3\right)} = \frac{3EI}{L^3} \]
Let's try a pinned-pinned column.

\[ P_{cr} = P \]

The moment equilibrium in the column at any point \( x \) is written as:

\[ M(x) = 0 = -P_{cr} \delta(x) + M(x) = 0 \]

\[ \delta(x) = \frac{w}{L} \]

\[ P_{cr} = P \text{ and } \delta(x) = \frac{w}{L} \]

\[ P_{cr} = \frac{\frac{w}{L}}{\frac{1}{2}EI} \]

Let \( P_{cr} = P \) and \( \delta(x) = \frac{w}{L} \)

For \( \delta(x) \) to be positive \( w \), the curvature is negative.

In order to solve for \( P = P_{cr} \), we must know the buckled shape \( w = \delta(x) \).

It turns out that the buckled shape is:

\[ w = a \sin \left( \frac{\pi}{L} x \right) \]

\[ a = \text{constant representing the maximum amplitude of deformation} \]

\[ \frac{dw}{dx} = a \cdot \cos \left( \frac{\pi}{L} x \right) \]

\[ \frac{d^2w}{dx^2} = -a \left( \frac{\pi^2}{L^2} \right) \sin \left( \frac{\pi}{L} x \right) \]

\[ P_{cr} = \frac{P}{\frac{1}{2}EI} \]

This is known as the elastic buckling load, or as the Euler buckling load (after Leonard Euler, 1757).
SOLUTION TO PROBLEM

An approximate solution can be obtained for this cantilever. By noting that the EA and PS are
concentrated at the joint, the stiffness of the struts in series can be
\[ F = (K_1 + K_2)E \]
\[ F = K_1E + K_2E \]

For one half:
\[ P_{ NI} = \frac{1}{2}L \]
\[ P_{ SI} = \frac{1}{2}L \]
\[ P_{ NI} = \frac{1}{2}L \]
\[ P_{ SI} = \frac{1}{2}L \]
SPRINGS IN SERIES (MORE FLEXIBLE = LOWER \( P_{cr} \))

\[ P_{cr1}^{1} + P_{cr2}^{1} = \frac{E}{k_{1}} + \frac{E}{k_{2}} = \frac{E(k_{1} + k_{2})}{k_{1}k_{2}} = \frac{E}{k_{1}k_{2}} \]

\[ P_{cr} = \frac{E}{k_{1} + k_{2}} = \frac{\frac{E}{k_{1}k_{2}}}{\frac{E}{k_{1}} + \frac{E}{k_{2}}} = \frac{E}{k_{1}} \left( \frac{1}{k_{2}} \right) = \frac{1}{k_{2}} \left( \frac{1}{k_{1}} \right) \]

SPRINGS IN PARALLEL \( k = k_{1} + k_{2} \) STIFFER

SPRINGS IN SERIES \( k = \frac{1}{k_{1} + k_{2}} \)

Since \( P_{cr} = kL \), \( P_{cr}'s \) CAN BE ADDED IN PARALLEL, OR IN SERIES JUST LIKE \( k's \)

\[ P_{cr1} = \frac{3EI}{L^{2}} \quad P_{cr2} = \frac{4.77EI}{L^{2}} \]

\[ P_{cr} = \frac{EI}{L^{2}} \left( \frac{1}{3 + \frac{1}{4.77}} \right) = \frac{2.33EI}{L^{2}} \]

\[ \frac{2.33 - 2.97}{2.97} \times 100 = \text{CONSERVATIVE BY 6.9%} \]
Stability

\( P_e = \frac{\pi^2 \varepsilon}{(KL)^2} \), \( P_f = AF_f \)

Inelastic Buckling

\( F_e = \frac{P_e}{A} = \frac{\pi^2 \varepsilon}{(KL)^2} = \frac{\pi^2 \varepsilon}{(KL)^2} \)

Elastic Buckling

\( \frac{F_f}{F_e} = \varepsilon A \), \( \lambda_c = \frac{KL \sqrt{F_f}}{\pi F_e} \)

Material Constants (Slenderness Parameter)

For Steel, \( F_f = 0.44F_f \), \( \lambda_c = 1.5 < \lambda_c \)


\( F_e \geq 0.44F_f \leq \frac{\pi^2 \varepsilon}{(KL)^2} \rightarrow KL \leq \frac{15 \pi F_f}{\sqrt{F_f}} = 4.71 \sqrt{\frac{F_f}{F_f}} \)

Inelastic Buckling

\( P_{cr} = (0.658) \frac{P_e}{F_f} A_3 \)

\( F_e < 0.44F_f \) or \( KL > \frac{4.71 \sqrt{F_f}}{F_f} \)

Elastic Buckling

\( P_{cr} = 0.877P_e \)

Note: 0.6(0.877) = 0.526 \( \approx \frac{12}{23} \)

For 50 ksi steel \( 4.71 \sqrt{\frac{\varepsilon}{50}} = 113 \)
For a column 20'-0" long:

\[ K \ell / \gamma = \frac{240}{2.02} = 119 \]
\[ P_y = \frac{50 \text{ksi} \times (323^2)}{2} = 16150 \text{kips} \]
\[ P_e = \pi^2 \left( \frac{2000}{16700} \right)^2 = 83000 \text{kips} \]
\[ P_y = 16150 \text{kips} = 0.195 \]
\[ P_e = 83000 \text{kips} \]
\[ P_{cr} = \frac{(0.658)^{0.195} (16150)}{14900} = 14900 \text{kips} \]
\[ \phi P_y = 0.9 (14900) = 13400 \text{kips} \]
\[ P_a = \frac{14900}{1.6} = 9200 \text{kips} \]

\[ K \ell / \gamma = \frac{240}{2.02} = 119 \]
\[ P_y = \frac{50 \text{ksi} \times (9.12^2)}{2} = 456 \text{kips} \]
\[ P_e = \pi^2 \left( \frac{2000}{371} \right)^2 = 184 \text{kips} \]
\[ P_y = 456 = 2.48 \]
\[ P_e = 184 \]
\[ P_{cr} = 0.877 (184\%) = 161 \text{kips} \]
\[ \phi P_y = 0.9 (161\%) = 145 \text{kips} \]
\[ P_a = \frac{161}{1.6} = 96 \text{kips} \]

(SEE P.4-21)
\[ P_{cr} = \frac{T^2 EI}{(KL)^2} \]

For a Pinned-Pinned Column.

Let the symbol \( K \) account for the effects of various boundary conditions.

\[ P_{cr} = \frac{T^2 EI}{(KL)^2} \]

\[ P_{cr} = \frac{T^2 EI}{(2.0L)^2} \]

\[ P_{cr} = \frac{T^2 EI}{(0.7L)^2} \]

\[ P_{cr} = \frac{T^2 EI}{(0.5L)^2} \]

Redraw the above four conditions such that \( KL \) is the same for each.

\[ KL \]

\[ L \]
K = 1.0
K = 2.0

BEAM ACTS AS ROTATIONAL SPRING

0.5 < K < 1.0
1.0 < K < 2.0
SECOND ORDER EFFECTS

\[ Pw + M_0 = -\frac{c^2w}{\alpha^2} EI \]

\[ P_0 \sin \theta + M_0 = -\frac{c^2 w}{\alpha^2} EI \sin \theta \]

\[ L = \frac{L^2}{L^2 - \theta} \]

\[ P = \frac{Pe_P}{Pe-P} \]

\[ \omega = \frac{P_0}{P_0} \]

\[ M = M_0 + P_0 < My \]

\[ = M_0 + \frac{PM_0}{Pe-P} = \frac{(Pe-P+P)M_0}{Pe-P} = \frac{PeM_0}{Pe-P} \]

\[ T_M = \frac{M_0}{Pe-P} = \frac{M_0}{1-P} < My \]

\[ \frac{M}{My} = \frac{M_0}{My(1-P/Pe)} \]

\[ \frac{P}{Pe} + \frac{M_0}{My(1-P/Pe)} = 1.0 \quad \text{Code has failed} \]

LRFD \[ \frac{P}{P_0} + \frac{8M_0}{9\phi My(1-P/Pe)} \leq 1.0 \]

ASD \[ \frac{P}{P_0} + \frac{8M_0}{9\phi M_0(1-1.6P/Pe)} \leq 1.0 \]

H1-1a p.16.1-70 + C2-1a p.16.1-21 + C2-2
LeMessurier Consultants
Structural Engineers

Subject: COLUMN DESIGN

3.5" LWC ON 46 psf
3" Zoga Deck 2 psf
HUNG 5 psf
FLR 2 psf
FFP1 + PONDING 15 psf
PARTITIONS 20 psf
90 psf

LINE OFFICE 50 psf
FACADE 20 psf

5 STORIES @ 14'-0" EA

INTERIOR COLUMN (I)

\[ A_I = (30')^2 = 900 \text{ sq ft} \]
\[ D = 0.09 \text{ (200)} = 81 \text{ k} \]
\[ L = 0.050 \text{ (900)} = 45 \text{ k} \]
\[ N_4 = 0.50 \]
\[ N_3 = 0.40 \]

PERIMETER (P)

\[ A_P = 480 \text{ sq ft} \]
\[ D = 43.2 \text{ k} \]
\[ L = 24 \text{ k} \]
\[ F = 0.02(14)(30) = 8.4 \text{ k} \]
\[ D = 57.6 \text{ k} \]
\[ N_4 = 0.60, N_3 = 0.50 \]
\[ N_2 = 0.50 \]
\[ N_1 = 0.40 \]

CORNER (C)

\[ A_C = 256 \text{ sq ft} \]
\[ D = 23.0 + 7.0 = 30 \text{ k} \]
\[ L = 128 \text{ k} \]
\[ N_4 = 0.72 \]
\[ N_3 = 0.6 \]
\[ N_2 = 0.5 \]
\[ N_1 = 0.4 \]

LINE LOAD REDUCTION

\[ I: 50 \text{ psf} \left( \frac{0.25 + 15}{\sqrt{4(256)}} \right) = 25 \text{ psf} \]
\[ P: 50 \left( \frac{0.25 + 15}{\sqrt{4(480)}} \right) = 30 \text{ psf} \]
\[ C: 50 \left( \frac{0.25 + 15}{\sqrt{4(256)}} \right) = 36 \text{ psf} \]

ROOF

\[ D = \text{ Roofing + Insulation + Roof Deck + Hung + FPM} = 25 \text{ psf} + 5 \text{ psf} = 30 \text{ psf} \]

\[ \text{SNOW} = 30 \text{ psf} + \text{DRIFT} \text{ (Assume no drift in this area)} \]
COLUMN SPlice BTW 3+4, 
SO N3 DRIVES COLUMN SIZE.

SIMPLIFY LINE LOAD REDUCTION:
I: \( N = 0.40 \)
P: \( N = 0.50 \)
C: \( N = 0.60 \)

### INTERIOR

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<th>ΣD+L</th>
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<td>27k</td>
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<td>18k</td>
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### PERIMETER

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<tr>
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<td>291k</td>
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### CORNER

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<tr>
<td>1</td>
<td>32k</td>
<td>8k</td>
<td>183k</td>
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</table>
Is NBx31 an appropriate minimum column size?

**Columns should also be designed to carry a minimum moment.** See "Section Tolerances" 7.13 p. 163, 42-163.53 in "Code of Standard Practice."

- \( 14' (12\%) = 0.336' \) 437 kips (0.336') = 12.6' = Design kips

**Tolerance for Out-of-plumb Column**

- Simple connections induce some moment into column. Assume \( C = 2.5' \), max girder load at \( = \frac{140 sf}{30'}(30) = 63k \) 12k.

- Apply minimum 5% to each column about weak axis.

Check available strength of W10 x 33

**Spec. Chart H, H1.1(a)**

- For \( P_c \geq 0.2 \)

\[
\frac{P_c}{P_{cr}} + \frac{8}{9}\left(\frac{M_{max}}{M_{cr}} + \frac{N_{max}}{N_{cr}}\right) \leq 1.0
\]
\[ W_{10 \times 33} \]

**Note:**
- \( Z_y = 14.0^3 \)
- \( M_{cy} = 35(14)/12 = 35.06' \)

**Design:**

- \( A = 9.71^2 \)
- \( I_y = 36.6''^4 \quad I_x = 171''^4 \)
- \( S_y = 9.20^3 \quad S_x = 35.0^3 \)
- \( M_{cy} = 33(120)/12 \quad M_{cx} = 33(35.0)/12 \)
- \( = 25.36' \quad = 76.36' \)

\[ P_y = \frac{987(2900)(36.6)}{(168)^2} = 371k \]
\[ P_x = 50080(9.71^2) = 486k \]
\[ P_{a} = 0.6(0.658)^{1.31}(486) = 167k \]

**Perimeter Under 4th Floor D + L = 163k**

\[ \frac{163k}{169k} + \frac{8}{9} \left( \frac{56''(0.6)}{25.36''(1 - 1.6(163k)/37k)} + \frac{56''(0.6)}{96.36'} \right) \]

\[ = 0.764 + 0.355 + 0.028 = 1.35 > 1.0 \]

**Try W_{110 \times 39}**

- \( A = 11.5^2 \)
- \( I_y = 45.0''^4 \quad P_{a} = 50(11.5) = 575k \)
- \( S_y = 11.8^3 \quad P_{ey} = 987(2900)(45.0) = 456k \)
- \( M_{cy} = 31.1k' \quad P_{a} = 0.6(0.658)^{1.26}(575) = 205k \)

\[ \frac{163}{204} + \frac{8}{9} \left( \frac{5(0.6)}{31.1(1 - 1.6(163)/456)} + \frac{5(0.6)}{116k'} \right) \]

\[ = 0.799 + 0.200 + 0.023 = 1.02 > 1.0 \]

**Try:**

\( Z_y = 17.2^3 \)
\( M_{cy} = 30(17.2)/12 = 43.6' \)

\[ \frac{163}{204} + \frac{8}{9} \left( \frac{5(0.6)}{43.6(0.428)} + \frac{5(0.6)}{116} \right) \]

\[ = 0.799 + 0.145 + 0.023 = 0.967 < 1.0 \quad OK \]
### Structural Steel Column Schedule

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<th>Level</th>
<th>Mark</th>
<th>B</th>
<th>D</th>
<th>C</th>
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**Type**
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(Note: These columns require some stabilizing system)

### Base Design (J8, p. 161-115)

### Spread Footings

\[
I = \frac{450k (2.5)}{1.7 (0.85) (96)} = 195''^2 = 14'' \times 14'' \text{ (min)}
\]

\[
f_c = 4.0k; \quad f_y = 86.6k
\]

\[
n = \frac{14'' - 0.8(10'')}{2} = 3''
\]

\[
\frac{450k}{196''^2} = 2.3\text{ ksi}; \quad 2.3\text{ ksi}(3'')^2 = 10.4k''
\]

\[
10.4k'' = \left(\frac{1''}{6k''}\right) \times \frac{2}{6}\sqrt{2.89''} = 1.7''
\]

\[
P = \frac{291k (2.5)}{1.7 (0.85) (4)} = 126''^2 = 12'' \times 12''
\]

\[
\frac{2}{2} = \left(\frac{20(2)'(1.6)}{36}\right) \times \frac{1}{6} = 1.06''
\]

For base size, see col sched

- Probably too tight for 110 column
- Consider placing anchor bolts outside
- 1 ½" col on larger base flange
CONSTANT PRESSURE ON LEELWARD SIDE

PRESSURE VARIES WITH HEIGHT ON WINDWARD SIDE

70'-0"

70'-0"

120'-0"
LONG DIRECTION

90'-0"
SHORT DIRECTION

WINDWARD

LEELWARD

30'-0"  30'-0"  30'-0"

1  2  3  4  5  R

0'-0"

0'-0"

0'-0"
ASCE 7-05

3 SECOND GUST C 33'
DIRECTIONALITY FACTOR
IMPORTANCE
EXPOSURE B

NO TOPOGRAPHIC EFFECTS
GUST FACTOR
SHAPE FACTORS

\[
V = 105 \text{ mph} \\
K_d = 0.85 \\
\theta = 1.0 \\
X = 7.0 \\
Z_g = 1200' \\
K_e = 2.01 \left( \frac{Z_g}{270} \right)^{0.5} \\
\text{or } 2.01 \left( \frac{15}{Z_g} \right)^{0.5}
\]

\[
V = 0.00256 K_e K_{ex} K_d Y^2 I 
\]

\[
P = q (G_{cp}) - q (G_{cp}) \quad \text{BE FULLY-ENCLOSED BUILDINGS}
\]

\[
P = (24 \text{ psi}) K_e (0.85)(0.8) \quad \text{WINDWARD} = (16.3 \text{ psi}) K_e
\]

\[
(24 \text{ psi}) K_h (0.85)(-0.5) \quad \text{LEEWARD} = (10.2 \text{ psi})(0.85) = 9.1 \text{ psi}
\]

<table>
<thead>
<tr>
<th>STORY</th>
<th>HEIGHT</th>
<th>( K_e )</th>
<th>( P_w )</th>
<th>( P_a )</th>
<th>( P )</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>56'</td>
<td>0.892</td>
<td>14.5</td>
<td>9.1</td>
<td>23.6</td>
<td>32k</td>
</tr>
<tr>
<td>4</td>
<td>42'</td>
<td>0.837</td>
<td>13.6</td>
<td>9.1</td>
<td>22.7</td>
<td>39k</td>
</tr>
<tr>
<td>3</td>
<td>28'</td>
<td>0.771</td>
<td>12.6</td>
<td>9.1</td>
<td>21.7</td>
<td>37k</td>
</tr>
<tr>
<td>2</td>
<td>14'</td>
<td>0.575</td>
<td>9.4</td>
<td>9.1</td>
<td>18.5</td>
<td>32k</td>
</tr>
</tbody>
</table>

\[
F = \left( \frac{120' \times 2'}{2} \right) \left( h_i + h_i \right) \frac{P}{1000} 
\]
ASCE 7-05

Boston  LAT = 42.4°  
      LONG = -71.1°  
Site Class D  

2002 USGS MAPS  
2% IN 50 YEAR EARTHQUAKE 
MCE EVENT  

<table>
<thead>
<tr>
<th>S_2</th>
<th>S_5</th>
<th>F_1</th>
<th>F_2</th>
<th>S_{min}</th>
<th>S_{max}</th>
<th>T_5</th>
<th>T_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.284</td>
<td>0.0685</td>
<td>1.57</td>
<td>2.4</td>
<td>0.446</td>
<td>0.164</td>
<td>0.3683</td>
<td>0.0743</td>
</tr>
</tbody>
</table>

T_a = 0.02 (70')^{0.75} \times 0.4845 > 0.3683 \quad (12.82)

I = 1.0

S_{SI} = \frac{2}{3} \times 0.164 = 0.109 \quad (11.44)

V = \frac{S_{SI}}{T_R} \quad (12.81)

W = SEISMIC WEIGHT

CONG+DECK = 48 psf
CARPET = 2 psf
PONDING = 5 psf
FRAMING = 10 psf
HING = 5 psf
PARTITIONS = 10.5 psf

FACADE = 20 psf

TRIBUTARY AREA
(92') (122') = 11200 sq ft

R = 0.03 \times 11200 = 336 k
Z-5 = 4 \times 0.08 \times 120 = 355 k
F = 0.02 \times (67) \times 120 = 577 k

W = 4500 k
Design Acceleration Response Spectrum (ARS)

\[ S_0 = 0.297 < 0.333 \]
\[ S_D = 0.109 < 0.138 \]

Seismic Design Category \( B \)

\[ R = 3.25 \text{ ordinary concentrically braced frame} \]

\[ k = 1.0 \]
\[ F_x = \left( \frac{w_k h_k}{\sum w_k h_k} \right) V \]
\[ V = \left( 0.109 \times \frac{4500}{0.484} \right) = 312k \]

<table>
<thead>
<tr>
<th>STORY</th>
<th>( w )</th>
<th>( h )</th>
<th>( w h^k )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>430k</td>
<td>70'</td>
<td>3,01k^4</td>
<td>54k</td>
</tr>
<tr>
<td>4</td>
<td>1020k</td>
<td>56</td>
<td>5,74k^4</td>
<td>103k</td>
</tr>
<tr>
<td>3</td>
<td>1020k</td>
<td>42</td>
<td>4,28k^4</td>
<td>77k</td>
</tr>
<tr>
<td>2</td>
<td>1020k</td>
<td>28</td>
<td>2,86k^4</td>
<td>52k</td>
</tr>
<tr>
<td>1</td>
<td>1020k</td>
<td>14</td>
<td>1,93k^4</td>
<td>26k</td>
</tr>
</tbody>
</table>

\[ \sum F = 312k \]

<table>
<thead>
<tr>
<th>STORY</th>
<th>( h )</th>
<th>( F_{h} )</th>
<th>( V_h )</th>
<th>( M_h )</th>
<th>( F_E )</th>
<th>( V_E )</th>
<th>( M_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>70'</td>
<td>32k</td>
<td>32k</td>
<td>448k^6</td>
<td>103k^6</td>
<td>157k^6</td>
<td>756k^6</td>
</tr>
<tr>
<td>4</td>
<td>56'</td>
<td>39k</td>
<td>71k</td>
<td>108k</td>
<td>208k</td>
<td>234k</td>
<td>2950k^6</td>
</tr>
<tr>
<td>3</td>
<td>42'</td>
<td>37k</td>
<td>108k</td>
<td>1440k^6</td>
<td>103k</td>
<td>157k^6</td>
<td>756k^6</td>
</tr>
<tr>
<td>2</td>
<td>28'</td>
<td>35k</td>
<td>143k</td>
<td>2750k^6</td>
<td>52k</td>
<td>286k^6</td>
<td>6230k^6</td>
</tr>
<tr>
<td>1</td>
<td>14'</td>
<td>32k</td>
<td>175k</td>
<td>4950k^6</td>
<td>26k</td>
<td>312k^6</td>
<td>1020k^6</td>
</tr>
</tbody>
</table>

7450k^6
LOAD COMBINATIONS

\[ \Delta_e = \frac{27(6)^3}{3EI} \]
\[ \Delta_e = \frac{27(6)^3}{3EI} \]
\[ \Delta_{Bo} = \frac{27(6)^3 + 27(5)^3(10)}{3EI} = \frac{V_B(16)^3}{3EI} \]
\[ V_B = 4500 \left( \frac{3}{3375} \right) = 46 \]

27k - 46 = 23k  
23/27 = 0.85

0.7(14) = 9.8 > 8
0.7(78) = 55 > 44
0.75(0.7)(24) = 127 > 123 (12, 13, 4)

Ordinary Concentrically Braced Frame
R = 3.25 "Limited Inelastic Deformations"

Elastic

Weak Beam

Bends

Inelastic

Low-Cycle Fatigue

Fracture Potential
AISC SEISMIC PROVISIONS SECTION 14

14.2 BRACING MEMBERS

MEET PROVISIONS OF 8.2.6 "SEISMICALLY COMPACT"

RECTANGULAR HSS \( \frac{L}{t} \leq 161 = 0.64 \sqrt{\frac{29,000}{46}} \)

\[ KL \leq 100 = 4 \sqrt{\frac{29,000}{46}} \]

\( K = 1.0 \)
\( L = 20.5' \times 20' \)

\( \gamma \geq 1.0 (20') (12") / 100 = 2.4" \)

ALLOWABLE BRACE SECTIONS

<table>
<thead>
<tr>
<th>Section</th>
<th>( \frac{b}{t} )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS 7x7 x 5/16</td>
<td>41.9 ft</td>
<td>12.1</td>
</tr>
<tr>
<td>7x7 x 5/32</td>
<td>50.6 ft</td>
<td>9.05</td>
</tr>
<tr>
<td>8x8 x 5/16</td>
<td>48.7 ft</td>
<td>14.2</td>
</tr>
<tr>
<td>8x8 x 5/32</td>
<td>59.1 ft</td>
<td>10.8</td>
</tr>
</tbody>
</table>

14.4 BRACING CONNECTIONS

\[ \frac{E_f A_g}{1.5} = 1.3 (460k) (11.6^\circ) = 462k \]

OR AMPLIFIED SEISMIC LOAD

\[ \Omega = 2.0 (0.7) = 1.4 \]

\[ 1.4 (10.7k) = 150k \leq 462k \]
14.3 SPECIAL BRACING CONFIGURATION REQUIREMENTS

**Post-buckling Compression Capacity**

\[ P_{nc} = 0.3 P_n = 0.3 \times 183k = 54.9k \]

**Tension Capacity**

\[ P_{nt} = \frac{P_n}{A_y} \text{ or System Capacity} \]

\[ \frac{15}{20.5} = 0.721 \]

\[ \frac{14}{20.5} = 0.682 \]

---

**Brace Size**

HSS 7 x 7 x 1/2 in

\[ P_n = 183k \quad \text{e20'} > 33k + 0.7(10k) = 108k \]

\[ P_n = 183 \quad \text{in} \]

\[ 0.6 \]

\[ 0.3 P_n = 0.3(183) = 55k \]

\[ 47k \]

\[ 40k \]

\[ 27k \]

\[ 97k \]

\[ 97k \]

\[ 570k \]

\[ 570k \]

\[ 570k \]

\[ 570k \]

\[ \frac{570k}{33k} = 17.3 \]

\[ \frac{570k}{33k} = 17.3''^3 \quad (\text{W27x84, } S_x = 213''^3) \]
8.3 Column Strength

Amplified Seismic Load \( S_2 = 2.0 \)

\[ 2.0 (0.7) = 1.4 \]

Design all braces as HSS 7x7x7/8

Designs gusset, beam, cols based on \( D + 1.4E \)

\[ 90(12) = 32.7^2 \times 3 \quad (H16 \times 26 \quad S_2 = 38.4^2) \quad 33 \]

\[ 270(12) = 98.2^2 \times 3 \quad (H24 \times 55 \quad S_2 = 114^2) \quad 33 \]

\[ 365(12) = 133^2 \times 3 \quad (H24 \times 68 \quad S_2 = 154^2) \quad 33 \]

\[ 495(12) = 180^2 \times 3 \quad (H27 \times 84 \quad S_2 = 218^2) \quad 33 \]

Check Uplift

\[ 0.6 (35k) - 0.75 (6^2 \times 243) = 236 k \]
TABLE 8-2

\[ f_p = 0.5 \times (0.6) \times (9000) = 21 \text{ kips} \]
\[ (5/16) \times (0.707) \times (2165) = 4.64 \text{ kips} \]
\[ (1/16) \times (0.707) \times (2165) = 0.928 \text{ kips} \]

\[ \frac{169 \text{ kips}}{4(4.64 \text{ kips})} = 9.1" = 10" \]
VECTICAL FORCES
167k (14) = 115k
115k + 9k = 124k
124k = 10.82 k
115w/m

MOMENTS
119k (7.2) = 857k
Fg (14.82 + 0.65 + 0.49) = 857 k
Fg = 12k

HORIZONTAL FORCES
169k (16) = 124k

BOLTS
(10.82 + 10.42) = 15.3k
≤ 21.2k o.k.

 Gusset PL WELD TO GIRDER

IGNE FOR MOMENT CALCS

DESIGN CLIPS FOR TRAY:
124k
4k
119k
1w7 GIRDER

BRACE - GIRDER - COLUMN CONNECTION

1' = 1'-0"
2 (169 k) (15') = 247 k
\( \frac{0.02 (247 k)}{20.5} = 5 k \)

3/4" φ SC BOLT IN SINGLE SHEAR
\( V_0 = 7.38 k \)

56 (18") = 706"
7.5 k (12" + 6") = 133.4" > 706" ok

Try 2 STIFF FRS
\( x \geq 3.75" 
\)
\( \frac{1}{2} \geq \frac{3.75}{6} \)

Use \( x = \frac{1}{2}" \)

STIFF FR
\( + \)

\( \frac{5k}{906"} \)

12" +

\( \frac{5k}{906"} \)

247/26 = 9.5 kfs
3347/256 = 134 kfs

\( A = 2 \times (5/6 \times 0.707 \times 5.9") = 26"^2 \)
\( S = 2 \times (5/6 \times 0.707 \times 5.9")/6 = 256"^3 \)

BEACE - GIEDEE CONNECTION
1" = 1' 0"
TYPICAL COLUMN SPLICES, TABLE 14-3, CASE VIII E
p.14-39

C7×12.25

NOTE: WELD COLUMN FLANGES
FIRST, REMOVE CHANNELS,
THEN WELD COLUMN WEB

3 BEACED FRAME
COLUMN SPlice
3’=1'-0"

FOR GRAVITY COLUMNS,
USE CASE VIII A, P.14-37
PJP WELD W/ E = 1/4" 
REPD. NO WEB WELD REPD.
LEAVE ERECTION CHANNEL IN PLACE.

TOP OF SLAB
\[ \frac{691k(2.3)}{1.7(0.86)(445)} = 299''^2 = 18'' \times 18'' \text{ min} \]

\[ 328k + 238k = 566k \]

\[ 134k = 9.1B = 10B \]
\[ 14.7 \text{ in.} \]
\[ \frac{1}{8}'' \text{ A307} \]

**FWDN DESIGN**

\[ 351k + 0.75(0.7)(243) = 479k \]

For \( f_o = 4k \text{sf} \), \( 479k \div 4k \text{sf} = 120''^2 = 11' \times 11' \)

**SLEDING**

\[ 0.75(0.7)(78k) = 41k (2 \times 44k \text{ wind}) \]
\[ 479k \times 0.25 = 120k > 41k \text{ oh} \]
CANTILEVERS

1. GEOMETRY + LOADS

REFERENCE FIBER

2. DEFORMED SHAPE

FREE BODY

3. MOMENT DIAGRAM
   - Moments drawn on tension side
   - REF FIBER in tension (+)
   - REF FIBER in compression (-)

4. SHEAR DIAGRAM
   - SHEAR DIAGRAM = SLOPE OF MOMENT DIAGRAM
   - View the fiber, NW REF FIBER ON BOTTOM, FROm LEFT TO RIGHT
   - INCREASING MOMENT, SHEAR = (+)
   - DECREASING MOMENT, SHEAR = (-)
   - REACTIONS + LOADS WILL APPEAR NW CORRECT ORIENTATION
   - IF SIGN CONVENTIONS ARE CORRECT

PARABOLIC

LINEAR

LOAD

REACTION

REACT FIBER IN COMPRESSION, MOMENTS ARE NEGATIVE

LINEAR B/C OF POINT LOAD

POSITIVE SHEAR B/C MOMENTS INCREASE FROM NEGATIVE TO ZERO FROM LEFT TO RIGHT

30 by 38 blocks at .25 inches
**Simple Beams**

- **Geometry** + Loads
- Deformed Shape
- Free Body
- 2-Point-Loaded Cantilevers
  - Moment Diagram
  - Shear Diagram

Most frame structures can be reduced to beams + cantilevers. Often, beams can be reduced further into cantilevers.

When you try to understand structures conceptually, look for the cantilevers inside the larger structure.
Fixed Beams

Points of Inflection

Cantilever

Simple Beam

Moment Diagrams
Same shape as simple span, but shifted up by a constant

Shear Diagrams
Exact same as simple span

Single Span Moment Diagram
Frames

Rigid

Pins

Max overturning

Zero overturning

Some rotation

Points of inflection between center and top

Beam shear = Axial forces in columns due to overturning
Fractures

Note: Moment + Shear diagrams not to scale

Greater slope

Greater shear

Not in equilibrium

Greater moment

Pior infl shifts on

Both legs have same slope

Legs have equal + opposite sides work equilibrium
THESE REACTIONS CANNOT EXIST
THE FRAME WILL
DEFORM SUCH THAT
THEY ARE NOT
NECESSARY.

CONSTANT MOMENT
HAS ZERO SLOPE
AND THEREFORE
ZERO SHEAR.

REACTION CANNOT EXIST.

NOTE SECOND ORDER
EFFECT ON BEAM +
EQU.: $M_2 > \frac{F}{2} \Delta$

$\Delta = \Delta_1 = \Delta_2$
BUILDING SYSTEMS

BRACED FRAME

FLEXURE

SHEAR

MOMENT FRAME

FLEXURE

SHEAR
SUBASSEMBLY

SYSTEM

MEMBER AXIAL FORCES RESEMBLE MOMENT + SHEAR DIAGRAMS (TENSION FORCES SHOWN)
**Elemental Deformations**

\[ \Delta = \frac{FL^3}{3EI} \]

\[ \theta = \frac{FL^2}{2EI} \]

\[ \frac{FL^2}{2EI} = \frac{ML}{3EI} + \frac{ML}{6EI} \]

\[ \frac{FL^3}{3EI} = \frac{ML}{3EI} (L) \]

**Deformed Shapes**

\[ \theta = \frac{ML}{2EI} \]

\[ \theta = \frac{ML}{3EI} \]

\[ \theta = \frac{ML}{6EI} \]

\[ \theta = \frac{ML}{6EI} \]
WORK = ENERGY

ENERGY IS CONSERVED: INTERNAL WORK = EXTERNAL WORK

ENERGY IS MINIMIZED: STRAIN ENERGY IS MINIMIZED

\[ \text{STRUCTURES DEFORM IN SUCH A WAY AS TO MINIMIZE INTERNAL STRAIN ENERGY} \]

\[ F \text{ AREA UNDER CURVE} = \text{WORK} = \text{STRAIN ENERGY} \]

EXT. WORK: \[ \frac{1}{2} Fx^2 = U \]
INT. ENERGY: \[ \frac{1}{2} kx^2 = U \]

\[ \frac{1}{2} Fx = \frac{1}{2} kx^2 \]

\[ F = kx \]

SOLUTION OBTAINED BY EQUILIBRIUM

SPRING DEFORMS UNDER LOAD F SUCH THAT STRAIN ENERGY IS MINIMIZED

FOR A GIVEN DEFORMATION, \( \delta \), FIND \( F(\delta) \)

TOTAL POTENTIAL ENERGY \( U = U = U \) ENERGY IS MINIMIZED OR MAXIMIZED WHEN \( U \) IS STATIONARY \( \delta U = 0 \)

\[ U = \int Fdx - \int F(\delta)dx \]

\[ \delta U = \int F(\delta)dx = \int (kx - F(\delta))dx = 0 \]
\[ k \Delta - F(\Delta) = 0 \]

\[ F(\Delta) = k \Delta \]

**Solution obtained by equilibrium**

**Axial**

\[ \Delta = \frac{FL}{EA} \]

\[ \sigma = \frac{F}{A} \]

\[ \varepsilon = \frac{F}{EA} \]

\[ F \Delta = \int_0^L \sigma \varepsilon \, dV \]

\[ \sigma = \frac{F}{A} \]

\[ \varepsilon = \frac{F}{EA} \]

\[ F \Delta = \int_0^L \int_A \frac{F^2}{EA^2} \, dA \, dx \]

\[ F \Delta = \int_0^L \frac{F^2}{EA} \, dx = \frac{FL}{EA} \]

**Flexural**

\[ \sigma = \frac{M(x) y}{I} = \frac{F x y}{I} \]

\[ \varepsilon = \frac{M(x) y}{EI} = \frac{F x y}{EI} \]

\[ F \Delta = \int_0^L \int_A (M(x))^2 y^2 \, dA \, dx \]

\[ F \Delta = \int_0^L \frac{F x^2}{EI} \, dx = \frac{FL^3}{3EI} \]

\[ \Delta = \frac{FL^3}{3EI} \]
**SHEAR**

\[ \tau = \frac{F}{A} \] 
\[ y = \frac{F}{G A} \] 
\[ F_d = \int_0^L \int_{A_x} \frac{F^2}{G A_x} \, dA \, dx \] 
\[ \Delta = \frac{F L}{G A} \]

**TORSION**

\[ \tau = \frac{T \rho}{J} \] 
\[ y = \frac{T \rho}{G J} \] 
\[ T_0 = \int_0^L \int_A \frac{\tau^2 \rho^2}{G J} \, dA \] 
\[ \theta = \frac{T L}{G J} \]

**FLEXURE**

\[ M_F = F x \] 
\[ M_{c x} = \frac{F x^2}{2} \] 
\[ F_d = \int_0^L \frac{F_e x}{2 E I} \, dx \] 
\[ \Delta = \frac{F L^4}{8 E I} \]
\[ F \Delta_F = \int_0^{L_1} \frac{F^2 x^2}{EI} \, dx + \int_0^{L_2} \frac{(F L_1)^2}{EI} \, dx \]

\[ \Delta_f = \frac{F L_1^3}{3EI} + \frac{F (L_2)^2 L_2}{3EI} \quad (1) \]

\[ \Delta_s = \frac{y_1 L_1^3}{3EI} + \left( \frac{1}{F} \right) \frac{v_2 L_2}{3EI} \quad (2) \]

\[ \Delta_s = \frac{y_1 L_1^2 (L_1 + L_2)}{3EI} \quad (3) \]

\[ F \Delta_s = \int_0^{L_1} \frac{F^2}{GA} \, dx + \int_0^{L_2} \frac{(F L_1)^2}{GA} \, dx \]

\[ \Delta_s = \frac{F L_1}{GA} + \frac{F (L_2)^2}{GA} \quad (1) \]

\[ \Delta_s = \frac{F L_1}{GA} + \frac{V_2}{F} \frac{L_2}{GA} \quad (2) \]

\[ \Delta_s = \frac{F L_1}{GA} \left( 1 + \frac{L_2}{L_1} \right) \quad (3) \]
\[ \Delta_s = \frac{10(Fh_2^2)}{FEA_h} L_h + \frac{10(FD)}{FEAD} L_D \]

\[ \Delta_s = \frac{(\Omega D)}{EA_h} FDL_D + \frac{FL_D^2}{LEA_h} \]

\[ \Delta_s = 2 \left( \frac{Fh_1 D}{FEAV} \right)^2 (1 + 2 + 3 + 4) \frac{h_1}{D} \]

\[ \Delta_s = \frac{2Fh_1^3 (30)}{EAV D^2} = \frac{Fh_1^3 (30)}{E2AV (\frac{D^3}{3})} \]

\[ \Delta_s = \frac{Fh_1^3 (30)}{125E2AV (\frac{D^3}{3})^2} = \frac{Fh_1^3}{4.2 EI} \]
\[ M_x = \frac{3}{8} qL^2 - \frac{9}{2} x^2 \]
\[ V_x = \frac{dM_x}{dx} = \frac{3}{8} qL - q \]
\[ M_{max} = \frac{qL^2}{64} - \frac{9}{128} L^2 = \frac{7}{128} L^2 = \frac{7L^2}{128} \]
\[ \Delta_0 = \frac{qL^4}{8EI} \]
\[ \Delta_1 = \frac{R_pL^3}{3EI} \]

Compatibility requires 
\[ \Delta_1 = \Delta_0 \]

\[ \frac{R_pL^3}{3EI} = \frac{3L^4}{8EI} \]
\[ R_p = \frac{3}{8} \frac{2L}{2} \]
\[ R_A = \frac{5}{8} \frac{2L}{2} \]
\[ M_B = \frac{qL^2}{2} - \frac{9L^2}{8} = \frac{7L^2}{8} \]
\[ M_C = \frac{9L^2}{8} - \frac{7L^2}{2(8)} = \frac{7L^2}{16} \]
VIRTUAL WORK

\[ \Delta \cdot \bar{T} = \int_{0}^{L} \frac{M(x) \bar{M}(x)}{EI} \, dx = \Delta \]

Although \( \bar{M}(x) \) is defined here to be linear, it could be any function.

\[ M(x) = \frac{\phi(x)}{EI} \]

\[ \Delta = \int_{0}^{L} M(x) \, dx \]

\[ \Theta = \int_{0}^{L} \frac{M(x)}{EI} \, dx \]

**IN GENERAL**

\[ \theta_{BA} = \int_{A}^{B} \phi(x) \, dx = \text{difference in tangent line slopes} \]

\[ \Delta_{BA} = \int_{B}^{A} \phi(x) \, dx \]
MOMENT AREA METHOD
AKA METHOD OF ELASTIC HEIGHTS
AKA CONJUGATE BEAM METHOD
AKA CURVATURE INTEGRATION

\[ \theta_B = \frac{FL (L/2)}{EI} = \frac{FL^2}{2EI} \]

\[ \Delta_B = \frac{FL^2 (L)(3L)}{3EI} = \frac{FL^3}{3EI} \]

\[ \theta_B = \frac{qL^3}{6EI} (L - \frac{1}{3}) = \frac{qL^3}{6EI} \]

\[ \Delta_B = \frac{qL^3}{6EI} (\frac{3L}{4}) = \frac{qL^4}{8EI} \]

TANGENT

PARABOLA

AREAS + CENTROIDS
\[ \Theta_A = \Theta_{BO} = \frac{FL}{4EI} \left( \frac{1}{2} \right) = \frac{FL^2}{16EI} \]

\[ \Delta_c = \frac{FL^2}{16EI} \left( \frac{3}{8} \frac{1}{2} \right) = \frac{FL^3}{48EI} \]

\[ \Theta_1 = \Theta_{B1} = \frac{M}{2EI} \left( \frac{1}{2} \right) = \frac{ML}{2EI} \]

\[ \Delta_c = \frac{ML^2}{8EI} \]

Compatibility requires:
\[ \Theta_{A1} = \Theta_A , \Theta_{B1} = \Theta_{BO} \]

\[ \frac{ML}{2EI} = \frac{FL^2}{16EI} \]

\[ M = \frac{FL}{8} \]

\[ \Delta_c = \frac{FL^3}{48EI} - \frac{FL^3}{64EI} = \frac{FL^3}{192EI} \]

Note: \[ \Delta_{c\text{fixed}} = \frac{1}{4} \Delta_{c\text{pinched}} \]
\[ \theta_0 = \theta_0 = \frac{2L^2}{8EI} \left( \frac{1}{2} \right) \left( \frac{1}{3} \right) = \frac{2L^3}{24EI} \]

\[ \Delta_c = \frac{2L^2}{8EI} \] \[ \text{ZERO SLOPE} \]

\[ M = \frac{9L^2}{12} \]

\[ \Delta_c = \frac{5L^3}{8EI} - \frac{7L^3}{96EI} = \frac{2L^3}{384EI} \]

\[ \Delta_C \text{INDO} = \frac{1}{5} \Delta C \text{Pinned} \]
\[ \Delta_0 = \frac{F(3L)^3}{48EI} - \frac{FL(\frac{L}{2})(L)}{4EI} - \frac{0.5FL^3}{16EI} \]
\[ \Delta_0 = \frac{27FL^3}{48EI} - \frac{FL^3}{16EI} - \frac{FL^3}{48EI} \]
\[ \Delta_0 = \frac{23FL^3}{48EI} \]

\[ \Delta_1 = \Delta_0 = \frac{5}{6} \frac{RcL^3}{EI} = \frac{23}{48} \frac{RcL^3}{EI} \]
\[ R_c = \frac{23}{48} F, \quad R_D = \frac{E - 23}{40} F \]

\[ \Delta_1 = \Delta_0 = \frac{5}{6} \frac{RcL^3}{EI} = \frac{23}{48} \frac{RcL^3}{EI} \]

\[ R_c = \frac{23}{48} F, \quad R_D = \frac{E - 23}{40} F \]
\[ \Delta_T = \frac{27FL^3}{48EI} - \frac{23}{24} \frac{23FL^3}{40EI} = \left(0.5625 - 0.55\right) \frac{FL^3}{EI} = 0.0115 \frac{FL^3}{EI} \]

\[ 0.0115 \frac{FL^3}{EI} = (0.552) \frac{FL^3}{48EI} \]

\[ K_T = \frac{2EI}{L} + \frac{3EI}{L} = \frac{5EI}{L} \]

\[ \Delta_T = \frac{FL^3}{48EI} - \frac{3}{40} \frac{FL}{EI} \left(\frac{1}{2}\right) \left(\frac{1}{4}\right) = \left(0.02053 - 0.009875\right) \frac{FL^3}{EI} = 0.0115 \frac{FL^3}{EI} \]
FORCE

\[ F = K \cdot \Delta \]  
**DEFORMATION**  
**STIFFNESS COEFFICIENT**  
*(STIFFNESS MATRIX)*  

\[ \Delta = \frac{1}{K} \cdot F = f \cdot F \]  
**FLEXIBILITY COEFFICIENT**  
*(FLEXIBILITY MATRIX)*  

**LINEAR ALGEBRA**

\[ [A] \{x\} = \{b\} \]

\[ [A] \] IS A MATRIX  
\[ \{x\} \] IS A VECTOR OF UNKNOWN QUANTITIES  
\[ \{b\} \] IS A VECTOR OF KNOWN QUANTITIES  

**SOLVE FOR**  
\[ \{x\} = [A]^{-1}\{b\} \]

**STIFFNESS METHOD** *(DISPLACEMENT METHOD)*

\[ \{\Delta\} = [K]^{-1}\{f\} \]

**SOLVE FOR DISPLACEMENTS**  
BASED ON STIFFNESSES + FORCES  

**FLEXIBILITY METHOD** *(FORCE METHOD)*

\[ \{f\} = [f]^{-1}\{\Delta\} \]

**SOLVE FOR FORCES**  
BASED ON FLEXIBILITIES + DISPLACEMENTS

\[ \Theta_A = \Delta_{BA} = \frac{ML^2}{L \cdot 3EI} = \frac{ML}{3EI} \]

\[ K = M \frac{\Theta_A}{L} = \frac{3EI}{L} \]

REFER TO L#11, P.45 FOR "ELEMENTAL DEFORMATIONS"
APPLY \( M_B \) TO POINT B SUCH THAT \( \theta_B \) BECOMES 0

\[
\theta_B = \frac{\Delta_{AB}}{L} = \frac{(ML/2)(L/3)}{6EI} = \frac{ML}{6EI}
\]

SYMPTOMATIC MOMENT IS UNKNOWN. REQUIRED TO CANCEL OUT \( \theta_B \) EXACTLY.

FROM P. 45, AT POINT OF APPLIED MOMENT, \( \theta = \frac{ML}{3EI} \)

THE FLEXIBILITY COEFFICIENT \( f = \frac{L}{3EI} \)

\[
M_B = \frac{\theta_B}{f} = \frac{ML}{6EI} \cdot \frac{3EI}{L} = \frac{M}{2}
\]

FOR \( M_B = \frac{M_A}{2} \)

\[
\theta_A = \frac{ML}{3EI} - \left(\frac{M}{2}\right)L = \frac{ML}{6EI} - \frac{ML}{4EI}
\]

\[
K = \frac{M}{\theta_A} = \frac{4EI}{L}
\]

\[
\theta_A = \theta_B = \frac{ML}{3EI} + \frac{ML}{6EI} = \frac{ML}{2EI}
\]

\[
K = \frac{M}{\theta_A \theta_B} = \frac{2EI}{L}
\]

\[
\theta_A = \theta_B = \frac{ML}{3EI} - \frac{ML}{6EI} = \frac{ML}{6EI}
\]

\[
K = \frac{M}{\theta_A \theta_B} = \frac{6EI}{L}
\]
**STIFFNESS METHOD**

\[ \begin{align*}
\theta_{10} &= \theta_{20} \\
K_{11} \Delta + K_{12} \theta &= \frac{F/2}{L} \\
K_{21} \Delta + K_{22} \theta &= 0 \\
\end{align*} \]

\[ K_{11} = \frac{12EI}{L^3} \]

\[ K_{12} = K_{21} = -\frac{6EI}{L^2} \]

\[ K_{22} = \frac{4EI}{L} + \frac{8EI}{L^2} = \frac{10EI}{L} \]

\[ \begin{bmatrix} 6/L^2 & -3/L \\ -3/L & 5 \end{bmatrix} \begin{bmatrix} \Delta \\ \theta \end{bmatrix} = \begin{bmatrix} F/2 \\ 0 \end{bmatrix} \]

\[ \det K = \frac{2EI}{L} \left( \frac{30}{L^2} - \frac{9}{L^2} \right) = \frac{42EI}{L^3} \]
\[ \Delta = \frac{5FL^3}{84EI} \]

\[ \theta = \frac{3FL^2}{84EI} = \frac{FL^2}{28EI} \]

\[ M_A = \frac{6EI}{L} \left( \frac{FL^2}{28EI} \right) = \frac{3FL}{14} \]

\[ M_B = \frac{FL}{4} + \left( \frac{FL}{4} - \frac{3FL}{14} \right) = \frac{7FL}{14} - \frac{3FL}{14} = \frac{4FL}{14} = \frac{2FL}{7} \]

\[ M_A + M_B = \frac{3FL}{14} + \frac{4FL}{14} = \frac{7FL}{14} \text{ OK} \]

Reduce to 1-DOF + apply moment distribution.
\[ \Delta_{10} = \frac{ML^2}{2EI} \]

\[ \Delta_{11} = \frac{2VL^3}{3EI} + \frac{VL^3}{EI} = \frac{5VL^3}{3EI} \]

\[ \Delta_{10} = \Delta_{11} \]

\[ \frac{ML^2}{2EI} = \frac{5VL^3}{3EI} \]

\[ V = \frac{3M}{10L} \]
\[ \frac{M}{2} + \frac{2EJL}{3} + \frac{4EI/L}{3} + \frac{6EI/L}{2} = \frac{10M}{4L} + \frac{6M}{5L} + \frac{3M}{4L} \]

Symmetry

\[ \frac{M}{6} + \frac{M}{3} + \frac{M}{14} \]

Anti-Symmetry
Δ_{10} = \frac{3PL}{16EI} \left( \frac{1}{2}L \right) = \frac{3PL^3}{32EI}

Δ_{11} = \frac{5VL^3}{3EI}

V = \frac{3PL^3}{32EI} \left( \frac{3EI}{5VL^3} \right) = \frac{9P}{160}

ASSUME RIGID COLUMNS

Δ_{20} = \frac{VL^3}{EI}

V = \frac{3P}{32}

\frac{3PL}{32} \quad \frac{3PL}{32} \quad \frac{3PL}{32} \quad \frac{3PL}{32} \quad \frac{VL/2}{32}
\[ \Delta_{20} = \frac{1}{6} (\frac{L}{2}) \left( \frac{3}{2} PL \right)(1 - \frac{1}{2})(L) = \frac{PL^3}{128EI} \]

\[ \Delta_{22} = \frac{V(L/2)(y_2)(L)}{3EI} = \frac{VL^3}{12EI} \]

\[ \Delta = \frac{3}{32} P \]

\[ 9PL = \frac{3}{64} \left( \frac{3PL + 3PL}{64} \right) \]

\[ \frac{3PL}{64} - \frac{3PL}{64} = \frac{3PL}{64} \]

\[ \Delta_{20} = \frac{P}{8EI} \left( \frac{1}{4} \right) \left( \frac{L}{2} \right) \left( \frac{L}{2} \right) = \frac{P^3}{128EI} \]

\[ V = \frac{12}{128} P = \frac{3}{32} P \]

\[ -\delta = \frac{1}{40} kL \]
Wind stiffness often controls the design of moment frames.

Code does not specify drift limits.

Rule of 12,000

\[ Y = \frac{4500 \text{ k}(1200 \text{ ft})}{(70 \times 120)(90)} = 6 \text{ psi} \]

\[ P = \frac{21 \text{ psf}}{12,000 + 6 \text{ psi} (90)} = \frac{h}{597} \]

\[ \Delta \leq \frac{14'(12')}{597} = 0.28'' \]

Simplify by adding to top of subassembly.

\[ V_c \Delta = \int^{84''} \frac{V_c z^2}{EI_c} \, dz + \int^{84''} \frac{V_c z^2}{EI_c} \, dz \]

\[ + \int^{180''} \frac{V_b z^2}{EI_b} \, dz + \int^{180''} \frac{V_b z^2}{EI_b} \, dz \]
\[ V_c \Delta = \frac{2 V_c^2 (84''^3)}{3 (29000) I_c} + \frac{2 V_c^2 (180''^3)}{3 (29000) I_b} \]

\[ V_b (30') = V_c (14') \]

\[ V_b = \frac{V_c (14')}{(30')} = \frac{V_c (7')}{(15')} = \frac{V_c (84'')}{(180'')} \]

\[ V_c \Delta = \frac{2 V_c^2 (84'')^3}{3 (29000) I_c} + \frac{2 V_c^2 (84'')^2 (180)}{3 (29000) I_b} \]

\[ \Delta = \frac{2 V_c (84'')^2}{3 (29000)} \left( \frac{84}{I_c} + \frac{180}{I_b} \right) \]

DESIGN COL + BM TO CONTRIBUTE EQUALLY TO \( \Delta \)

\[ \frac{84}{I_c} = \frac{180}{I_b} = \frac{(0.14'') (3) (29000)}{2 (84'')^2 (36k)} = 0.0240 \]

\[ I_c = \frac{84}{0.024} = 3500''^4 \]

\[ I_b = \frac{180}{0.024} = 7500''^4 \]

\[ \frac{W114 x 283}{I_x} = 3840''^4 \]

\[ \frac{W36 x 135}{I_x} = 7800''^4 \]
\[ 36k(7') + 44k(14') = \frac{27k}{30'} \]

\[ 0.14'' = \frac{36(84)}{3(2900)} + \frac{44(63)}{3(2900)} Ic \]

\[ Ic = 18880''', \quad (2114 + \frac{730}{3}) = 14300''', \quad 13700''' \]

\[ 730''(21) = 15330'' \]

\[ 311''(21) = 6531'' \]

\[ +150''(30') = +4500'' \]

\[ 11031'' < 15330'' \]

**Design a Grade Beam**

\[ 44k(18')^3 = \frac{0.067''}{3(2900)(16320)} \]

\[ 44k(18')^3 = 4553'''' \]

\[ I_B = \frac{2(103)^3(18)^3}{44k(3)(2900)(0.071)} = 4470'''' \]
CONSERVATIVE TO ASSUME HIGHER STORY SAFETY AT BOTH INCLINATION POINTS

Determine upper col.

\[ \Delta_y \leq 0.28'' = \frac{24(L_y^2)}{3(2900)} \left( \frac{180}{T_c} + \frac{180}{T_0} \right) \]

\[ 84 = \frac{180}{T_c} = 0.14(3)(2900) = 0.082 \]

\[ I_c = \frac{T_0}{2(274)(84)^2} = 164 \times 2 \times 11 \]

\[ I_{by} = 180/0.032 = 5625''^4 \]

\[ 2(84)^2(180) \]

\[ I_{b2} = 2(446)(84^2)(180) \]

\[ = 279.5''^4 \]

\[ I_{b3} = 2(84)^2(180)^3 \]

\[ = 225''^4 \]

\[ 0.28 - 0.082 \]

\[ 3(2900) \]

30 by 38 blocks at .25 inches
\[ \Delta f = \frac{2(44k)(66')^2(39')}{{3}(2900lbs)(4330''')} + \frac{2(20.5')^2(171')^2(180'')}{{(44k)(3)(2900lbs)(7040'')}} = 0.085'' + 0.128'' = 0.213'' \]

\[ \Delta s = \frac{2(44k)(66')}{(161lbs)(241'')} + \frac{2(20.5')^2(171')}{(44k)(1200lbs)(224'')} + \frac{(161lbs)(36'')}{(44k)(1200lbs)(241'')} = 0.022'' + 0.013'' + 0.079'' = 0.114'' \]

\[ \Delta f + \Delta s = 0.213'' + 0.114'' = 0.327'' = \frac{1}{5}'' \]

\[ V_0 = \frac{446(14')}{(80')} = 20.56 \]

\[ Z = \frac{9040''}{4330''} \]
CENTRELINE MODEL

\[ \Delta = \frac{2(44\text{ ft})(84)^3}{5(2900)(4800)} + \frac{2(90.5\text{ ft})(90)^3}{(90\text{ ft})(2900)(90)^2} \]

\[ = 0.138'' + 0.142'' = 0.28'' = \frac{1}{600} \]

\[ \frac{0.28''}{0.327''} = 0.86 \]

*Model under predicts flexibility by 14%*

CHECK STRENGTH

EVALUATE SEISMIC FORCES

\[ T_q = 0.028(70')^{0.8} = 0.838 \]

\[ S_{D1} = 0.1093 \]

\[ W = 4500\text{ lb} \]

\[ R = 3.5 \text{ ordinary steel moment frames} \]

\[ V = \frac{0.1093(4500)}{3.5(0.838)} = 1676 < 1756 \text{ wind force} \]
<table>
<thead>
<tr>
<th>STORY</th>
<th>Vc</th>
<th>Mc</th>
<th>M_ac</th>
<th>V_d</th>
<th>M_d</th>
<th>M_d_b</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>18k</td>
<td>126 k'</td>
<td>930 k'</td>
<td>8.4k</td>
<td>126 k'</td>
<td>5866 k'</td>
</tr>
<tr>
<td>4</td>
<td>27k</td>
<td>187 k'</td>
<td>930 k'</td>
<td>12.6k</td>
<td>187 k'</td>
<td>9876 k'</td>
</tr>
<tr>
<td>2</td>
<td>36k</td>
<td>2.52 k'</td>
<td>1390 k'</td>
<td>16.8k</td>
<td>252 k'</td>
<td>12106 k'</td>
</tr>
<tr>
<td>1</td>
<td>44k</td>
<td>308 k'</td>
<td>1390 k'</td>
<td>20.5k</td>
<td>308 k'</td>
<td>1390 k'</td>
</tr>
</tbody>
</table>

**DESIGN END COLUMNS**

**2ND FLOOR**

\[
0.28'' \geq \frac{(22.5)(180)^3}{(22)(3)(2900)(9640)} + \frac{2(22)(84)^3}{3(29000)}I_c
\]

\[
I \geq \frac{2(22)(84)^3}{(0.28 - 0.142)(3)(29000)} = 272''^4
\]

\[
W14 \times 193 \text{ has } 2400''^4
\]

**4TH FLOOR**

\[
0.28'' \geq \frac{(36)(190)^3}{(14k)(5)(29000)(5700''^4)} + \frac{2(14k)(84)^3}{3(29000)}I_c
\]

\[
I \geq \frac{2(14)(84)^3}{(0.28 - 0.137)(5)(29000)} = 1334''^4
\]

\[
W14 \times 120 \text{ has } 1335''^4
\]
\[ 1.28'' = \frac{70''(12\text{ in})}{656} \]

**Weight of Moment Frame**

**BEAMS**

- 40' (60' in)
- 84' (60' in)
- 118' (60' in)
- 135' (60' in)
- 150' (60' in)
- 106' (60' in)

\[ \text{Total} = 38,169 \text{#} \]

**COLUMNS**

- 311' (32.5' in)
- 211' (37.5' in)
- 209' (32.5' in)
- 2(108' x 37.5' in)

\[ \text{Total} = 39,565 \text{#} \]

\[ 38,169 + 39,565 = 77,734 = 37\# \]

Wind load = 9.3\#
NOTES:
(1) THE BEAM TO COLUMN CONNECTION SHALL DEVELOP THE NOMINAL PLASTIC STRENGTH OF THE BEAM UNLESS NOTED (M=XX k') ON PLAN.
(2) SHEAR CONNECTIONS SHALL BE DESIGNED FOR THE END REACTIONS GIVEN ON THE DRAWINGS.
(3) FILLER METAL USED IN ALL FULL- Penetration Welds shall have a minimum Charpy V-Notch Value of 20 ft-lb at 20°F and 40 ft-lb at 70°F.
(4)Prime backing bar, backgauge, and re-weld bot flange, typical.
NOTES:

1. TOP OF STRUCTURAL SLAB SHALL BE AT ELEVATIONS NOTED ON KEY PLAN. SLAB'S SLOPE AT RAMPS. TOP OF STRUCTURAL STEEL SHALL BE AT ELEVATIONS NOTED ON KEY PLAN. STRUCTURAL STEEL FRAMING IN THE RAMPS SHALL SLOPE WITH THE RAMP. BEAMS SHALL HAVE TOPS FLUSH.

2. SLAB CONSTRUCTION SHALL CONSIST OF 4 1/2" NORMAL WEIGHT CONCRETE ON 2" DEEP GALVANIZED COMPOSITE STEEL DECK. TOTAL THICKNESS = 6 1/2". REINFORCE SLAB WITH 6X6 - .060 WW 12" PITCH PLUS ADDITIONAL REINFORCEMENT OF #6@24" CONT. IN SPAN DIRECTION OF STEEL DECK.

3. FOR GENERAL NOTES AND ABBREVIATIONS SEE DRAWING S-001. FOR TYPICAL DETAILS SEE DRAWING S-002 TO S-004.

4. FOR COLUMN SCHEDULE AND DETAILS SEE DRAWING S-204 AND S-205.

5. BR-1, ETC., INDICATES LATERAL BRACING LOCATIONS, FOR ELEVATIONS MEMBERS SIZES, AND AND DETAILS SEE DRAWING S-204 THRU S-207.

6. INDICATES SPAN DIRECTION OF STEEL DECK.

7. Indicates moment connection. See details on drawing S-004.

8. Indicates column stops.

9. Indicates column starts.

DEAD LOADS

CONC = 150 psf x (6/7) = 69 psf
      (145 psf) / 12" = 12 psi

DECK = incl w/alc (2.5 psf)

FRT = AS CALCULATED

HUNG = 5 psf

WET WEIGHT = 74 psf

SUPERIMPOSED DL = 6 psf

LIVE = 50 psf

PRELIMINARY DESIGN

\[ q = 129 \text{ psf} \times (7.6 \text{ psf})^{0.8} = 0.980 \text{ psf} \]

\[ f = 0.75 \times \frac{(29.7)^{2}}{8} = 1036 \text{ psi} \]

\[ S = 1036 \times \left(\frac{12}{24}\right) / 50 \text{ ksi} = 37.5 \text{ ksi} \]

\[ Z = 1036 \times \left(\frac{12}{24}\right) / 50 \text{ ksi} = 41.3 \text{ ksi} \]
SERVICEABILITY (MINIMUM STANDARDS)

\[ \Delta w = \text{WET WEIGHT} < \frac{1}{2} \times 80 \]
\[ \Delta \text{LT} = \text{LONG TERM: SUPERIMPOSED ADJACENT + \frac{1}{4} \text{LINE}} < \frac{1}{4} \times 80 \]
\[ \Delta \text{ST} = \text{SHORT TERM: \frac{3}{8} \text{LINE}} < \frac{1}{4} \times 40 \]
\[ (\text{LINE} < \frac{1}{8} \times 60) \]
\[ \Delta \sigma = \Delta \text{w} + \Delta \text{LT} + \Delta \text{ST} - C (\text{CANTILEVER}) < \frac{1}{3} \times 80 \]

PRELIMINARY SERVICEABILITY CHECK

\[ 29'(12')/280 = 1.24' \]

\[ \frac{5(1036')(27')^3(178.3')^3}{48(29000lbs')(1.24')} = 434''^4 \]

TRY A COMPOSITE BEAM

\[ L = 29' = 7.25' < 7.6' \]
\[ 4' \]
\[ A_c = \frac{7.25/12'}{}(4.5') = 392''^2 \]
\[ f_c = 41500 \]
\[ 0.85(41500')(392''^3) = 13336 \]
\[ W14x22 \text{ has } A_s = 6.49''^2 \]
\[ f_y = 50ksi \]
\[ 50ksi(6.49''^2) = 325.6 < 13336 \]

SPEC. CHP. I

(READ SPEC. + COMMENTARY THOROUGHLY!)

COMPOSITE ACTION IS CONTROLLED BY STEEL OR BY STUDS

\[ f_c = 5.7\sqrt{41500} = 36000lbs; \quad n = \frac{29000}{3600} = 8 \]
\[ A_e = 392''^2/8 = 49''^2 \]

FOR LONG-TERM LOADS, DEFLECTIONS DUE TO CREEP ALONE CAN BE AS MUCH AS TWICE THE ELASTIC DEFLECTION:

\[ A\text{LT} = \frac{\frac{49''^2}{3}}{3} \]

* SEE ACI 318-05 9.5.2.5 \[ \lambda_A = \frac{\xi}{\xi + 50p'} \]

FOR \( p' = 2.0 \) (LOAD DURATION OF 5 YEARS OR \( 9/1000 \)), AND \( p' = 0 \) (CONSERVATIVE), \( \lambda_A = 2.0 \)
LeMessurier Consultants
Structural Engineers

Subject: CEE-24
Composite Beams/Gravity Loads

| ENAST | SEC | A | y | Ay² | I | I
|-------|-----|---|---|-----|---|---
| CONC  | 49"² | 13" | 83"² | 83" | 166"⁴ | 988"⁴
| W     | 6.49"² | 9.8" | 623"² | 199" | 822"⁴ | 988"⁴
| S₂₁₂ = 57.2"³ | 16.7" |

| ENALT | SEC | A | y | Ay² | I | I
|-------|-----|---|---|-----|---|---
| CONC  | 16"² | 13" | 3.2" | 27" | 199"² | 68"² |
| W     | 6.49"² | 7.9" | 405"² | 199" | 68"² | 68"² |

Note: pp. 3-190-3-206 give values for I lb.

F₀ = 0.66(50 kips) = 33.3 kips;

P₀ = 33.3 kips (57.2"³)/12" = 163 k' > 103 k' ok

\( q_{lt} = \left( 5000 + \frac{1}{6} (50 \text{kips}) \right) 7.6' = 165 \text{ kips/foot} \)

\( A_{lt} = 5 \left( 0.165 \text{ kips/ft} \right) \left( 29 \right)^{\frac{3}{2}} = 5.133'' = 0.133'' = 0.262 \text{ in} \)

\( q_{st} = \frac{9}{16} \left( 50 \text{ kips} \right) 7.6' = 0.253 \text{ kips/foot} \)

\( A_{st} = 0.133 \left( 0.253 \right) (682) = 0.141'' = 0.247 \text{ in} \)

\( q_{w} = (74 \text{ psi}) 7.6' = 0.562 \text{ kips/foot} \)

\( M_{w} = (0.562 \text{ kips})(29)^{2} \approx 59 \text{k}' < M_{Q} = 80 \text{k}' (29 \text{ kips})/12'' \) ok

\( \Delta w = 5 \left( 59 \text{k}' \right) (29)^{2} (1728) = 1.55'' = 1.55 \text{ in} \)

30 by 38 blocks at .25 inches
The page contains a structural engineering analysis for composite beams with gravity loads. The calculations involve beam analysis with formulas and values including:

- Moment calculations: \( M_n = 325k \cdot (6.15 + 6.85) - 2(53.5k \cdot 6.15 + 0.17) / 12 = 291k \)
- Moment reduction: \( \phi M_n = 0.9(291k) = 262k \)
- Span calculation: \( 208 + x = 325 - x \)
- Characteristic load: \( 58.5k = 0.234'' \)
- Code load: \( 275 = 284 \)
- Inelastic load: \( 260k' = 267k' \)

**Typical Loads**

**Uniform Loads**
- Residential: 40 psf
- Office: 50 psf
- Retail: 100 psf
- Public Space: 120 psf
- Corridor: 80 psf
- Mechanical: 150 psf
- Sidewalk: 250 psf / AASHTO

**Snow, Water, Etc.**
Δx = 1.55" + 0.133" + 0.141" = 1.82" = ½" NG

Camber for wet weight

Notes: Min length for camber ≥ 25' 
Min camber ≥ ½"

Min increment ≥ ½"

Camber for 0.75 - 0.8 Δw

\[
\frac{0.75}{72} \cdot 1.24" = 1.18" = ½\]

Try W12 x 19

\[
A_t = 5.57\text{"}^2 \\
I = 130\text{"}^4 \\
S = 2.3\text{"}^3 \\
E = 24.7\text{"}^3
\]

Check gstr. Loads (204 k)

\[
\frac{72}{72} \cdot 32.3\text{k} = 41.3\text{k} \text{; NG}
\]

W14 x 22 is the lightest possible beam

Note: Shear connector strengths are affected by direction of deck, # studs/ton, position in ELS

Shear connectors (P.3-207 gives values for Qn)

\[
\text{Min 26% composite action} \\
Q_a = 0.25 \cdot 0.44\text{"}^2 \cdot 14\text{k} / 3600\text{k} = 13\text{k} \\
E = 13\text{k}
\]

Min studs e 2'-0" = \( \frac{2.9 + 1}{2} = 16 \text{ studs} = 50\% \text{ composite action} \)

Note: Min 50% composite action is recommended by deflections

To activate studs are not as large (comment/13.26)

P. 201/100

\[
\text{Check } M + Δx \\
I_{efl} = 199"^4 + \sqrt{325} (682-199) = 585"^4
\]

\[
f_c = \frac{157.5k}{12"} = 23.8 \\
\text{I}_{eff} = (199"^4 + 0.8(988-199)) \cdot 0.75 = 623"^4
\]

\[
f_c = 23.8\text{k} + 8.9\text{k} = 32.7\text{k} < 63\text{k}
\]

\[
\Delta x = 1.56" + 0.133(\frac{585}{682}) + 0.141(\frac{623}{788}) = 1.125" = 0.63" = ½\text{"} 0.63\text{"}
\]
Homework Assignments

CEE-24: Steel Design
**Subject**: BEAMS + FRAMES

1. Select a size for this girder based on the following strength and stiffness requirements:
   - Allowable flexural stress: 
     \[ 0.66 \times 50 \text{ kips} = 33 \text{ kips} \]
   - Allowable maximum displacement: 
     \[ \Delta_{\text{max}} \leq \frac{5}{360} = 0.68" \]
   - Obtain exact and approximate values for \( \Delta_{\text{max}} \) and compare the two answers.

2. Perform the same exercises as required in Prob #1.
   - Check shear, displacements.

3. What effect does the height of the girder have on Prob #2?

4. Perform the same exercises as required in Prob #1.

5. Assume the girder and cols have the same sizes as the girder in Prob #4. Obtain exact and approximate moment diagrams for this two-hinged frame.

6. Obtain exact and approximate vertical displacements for the frame in Prob #5.
Q1-0.4 SELECT A TYPICAL BEAM - GIRDER
Δ < 1/600 SPANDEL BEAM
Δ < 1/2" SPANDEL GIRDER
EXCAVE LIT = 40 psf
1'-0" PERIMETER SLAB OVER HANG

1. SELECT A TRANSFER GIRDER.

2. DESIGN A 60" DEEP PLATE GIRDER.

3. DESIGN A SPANDEL PLATE GIRDER.

4. NOT REQUIRED.

5. SELECT A CANTILEVER + DRAW ITS CONNECTION TO COLUMN B-3 AT 1/8" SCALE.

6. NOT REQUIRED.

7. DEVELOP DETAILS FOR THE TRANSFER IN PROBLEM #1 (SL B-2 4TH FLR).

8. DEVELOP A SET OF PLANS, SECTION MARKS, AND DETAILS BASED ON YOUR WORK IN PROB # 0 - PROB # 7.

NOTE: DOUBLE-CIRCLED NUMBERS ON THIS PAGE REFER TO NUMBERS LISTED ON PLANS P.2/2.
NOTES:
1. [ ] - COLUMN STARTS
2. [ ] - COLUMN STOPS
3. [ ] - SIMPLE CONN
4. [ ] - MOMENT CONN
5. [ ] - GIRDER UNDER COL

FLOORS 5 & 6
(Roofline Config, Diff Sizes)

SCALE: 1" = 40' 0"

4TH FLO

3RD FLO

2ND FLO

1ST FLO
1. Develop a column schedule for HN #2 for Cols A-1 through C-3. Draw at 1/2" = 1'-0" scale. Include base plate sizes and show all work.

2. Draw typical base fl + col splice details at 1/2" = 1'-0" scale. (10 ps ea.)

3. Detail welds for T2 girders 2 + 3.
   Develop a T2 girder schedule and typical detail (1/2" = 1'-0" scale)

All drawings must be to scale + engineering quality.

4. Provide explicit calculations for Cols B-1 + C-3 betw 1st + 3rd floors.

5. Design locations 4 + 6 from HN #2. Assume 3673# wind load.
LeMessurier Consultants
Structural Engineers

Subject: CEE-24

Braced Frames

Notes:
1. 3 1/4" 3000 psi
LC on 3" 20G
2. Galv. Composite
3. Steel Deck, Total
4. Slab Thickness = 6 1/4"
5. Reinf W/ HRF 6x6 - W12.4xW2.4
6. BR-2, BR-1
7. braces
8. Stabilizing beam
9. 1x0" slab
10. Overhang TYP

 Loads
Dead = wt of strct
10 psf hanging
20 psf flat
Facade = 25 psf
Wind < 1/2"
LNE = 50 psf
Office

Wind = ASCE 7-05
Exp B
G = 0.35
Seismic = ASCE 7-05
Site class D
(1) Design a typical beam, girder, spandrel beam, and spandrel girder.

(2) Develop story shear and overturning moment curves for wind + earthquake loads.

(3) Develop sketches for one braced frame showing relevant design forces for braces, beams + columns under wind + one under seismic loads.

(4) Design braces, beams + girders for wind, design braces, beams + girders as OC BF for seismic.

(5) Develop details for the three locations shown on the BR-1 elevation.
Draw deformed shapes, moment diagrams* (on the tension side), shear diagrams* (with arrows), and reactions for the following structures.

* Must be hardline drawings.

Account for axial deformation in pinned members.

Pins (M = 0)
CALCULATE THE VERTICAL DISPLACEMENT AT POINT A FOR EACH OF THE FOLLOWING STRUCTURES USING AT LEAST TWO METHODS. C/SE AT LEAST ONE STIFFNESS-BASED METHOD AND AT LEAST ONE FLEXIBILITY-BASED METHOD.

ACCOUNT FOR AXIAL DEFORMATION IN TEUSS MEMBERS.
Design a free-standing canopy 50'-0" long x 12'-0" wide x 10'-0" high for an MBTA Trolley Platform.

Snow = 30 psf
Wind = -21 psf, suction on roof

EQ = 0.3 g

Canopy must have sufficient strength to carry 60 psf live load.

Under snow, canopy must have sufficient stiffness for:

\[ \Delta = \frac{1}{240} \]

Note: Uniform load on entire area may not be the worst load case.

Draw:
(1) Plan(s)
(2) Front elevation
(3) Side elevation
(4) At least 4 of the most important details

Sketch:
(1) Axon/ISO
(2) Perspective
For Problems 1 - 4: A. Approx deformed shapes, moment diagrams + shear diagrams. Provide numerical approximations of internal force diagrams + of Δ at Pt A (Probs 1+2) and at Pt C (Probs 3+4).

B. Exact soln by flexibility method.
C. Exact soln by stiffness method.
D. Exact soln by computer.

5. Design a structure that encloses a minimum clear space of 300' x 126' x 73'.