Analytical model for coherent hemodynamics spectroscopy (CHS)

The CHS model describes sinusoidal oscillations at a given angular frequency ω. Oscillatory quantities are represented by phasors that are indicated in bold face. The model expressions for **O**(ω), **D**(ω), **T**(ω) (i.e. the phasors that describe the oscillations of oxy-, deoxy- and total hemoglobin concentrations) as a function of **cbv**(ω), **cbf**(ω), and $\mathbf{cmro_2}$(ω) (i.e. the phasors that describe the oscillations of cerebral blood volume, blood flow, and metabolic rate of oxygen) are as follows:

$$\mathbf{O}(\omega) = \mathrm{ctHb}\left[S^{(a)}\mathrm{CBV}_0^{(a)}\mathbf{cbv}^{(a)}(\omega) + S^{(v)}\mathrm{CBV}_0^{(v)}\mathbf{cbv}^{(v)}(\omega)\right] +$$
$$+\mathrm{ctHb}\left[\frac{\langle S^{(c)}\rangle}{S^{(v)}}\left(\langle S^{(c)}\rangle - S^{(v)}\right)\mathrm{F}^{(c)}\mathrm{CBV}_0^{(c)}\mathcal{H}_{RC-LP}^{(c)}(\omega) + \left(S^{(a)} - S^{(v)}\right)\mathrm{CBV}_0^{(v)}\mathcal{H}_{G-LP}^{(v)}(\omega)\right][\mathbf{cbf}(\omega) -$$
$$\mathbf{cmro_2}(\omega)]$$

$$\mathbf{D}(\omega) = \mathrm{ctHb}\left[\left(1 - S^{(a)}\right)\mathrm{CBV}_0^{(a)}\mathbf{cbv}^{(a)}(\omega) + \left(1 - S^{(v)}\right)\mathrm{CBV}_0^{(v)}\mathbf{cbv}^{(v)}(\omega)\right] +$$
$$-\mathrm{ctHb}\left[\frac{\langle S^{(c)}\rangle}{S^{(v)}}\left(\langle S^{(c)}\rangle - S^{(v)}\right)\mathrm{F}^{(c)}\mathrm{CBV}_0^{(c)}\mathcal{H}_{RC-LP}^{(c)}(\omega) + \left(S^{(a)} - S^{(v)}\right)\mathrm{CBV}_0^{(v)}\mathcal{H}_{G-LP}^{(v)}(\omega)\right][\mathbf{cbf}(\omega)$$
$$- \mathbf{cmro_2}(\omega)]$$

$$\mathbf{T}(\omega) = \mathrm{ctHb}\left[\mathrm{CBV}_0^{(a)}\mathbf{cbv}^{(a)}(\omega) + \mathrm{CBV}_0^{(v)}\mathbf{cbv}^{(v)}(\omega)\right],$$

where $\mathcal{H}_{RC-LP}^{(c)}(\omega)$ and $\mathcal{H}_{G-LP}^{(v)}(\omega)$ are the complex transfer function given by:

$$\mathcal{H}_{RC-LP}^{(c)}(\omega) = \frac{1}{\sqrt{1+\left(\frac{\omega t^{(c)}}{e}\right)^2}} e^{-i\tan^{-1}\left(\frac{\omega t^{(c)}}{e}\right)}$$

$$\mathcal{H}_{G-LP}^{(v)}(\omega) = e^{-\frac{\ln 2}{2}\left[\omega\, 0.281\left(t^{(c)}+t^{(v)}\right)\right]^2} e^{-i\omega\, 0.5\left(t^{(c)}+t^{(v)}\right)}$$

We have set $\mathbf{cbv}^{(c)}(\omega) = 0$ because of the negligible dynamic dilation and recruitment of capillaries in brain tissue. Because of the high-pass nature of the cerebral autoregulation process that regulates cerebral blood flow in response to blood pressure changes, we consider the following relationship between **cbf** and **cbv**:

$$\mathbf{cbf}(\omega) = k\,\mathcal{H}_{RC-HP}^{(\mathrm{AutoReg})}(\omega)\mathbf{cbv}(\omega) = k\,\mathcal{H}_{RC-HP}^{(\mathrm{AutoReg})}(\omega)\left[\frac{\mathrm{CBV}_0^{(a)}}{\mathrm{CBV}_0}\mathbf{cbv}^{(a)}(\omega) + \frac{\mathrm{CBV}_0^{(v)}}{\mathrm{CBV}_0}\mathbf{cbv}^{(v)}(\omega)\right],$$

where k is the inverse of the modified Grubb exponent, $\mathcal{H}_{RC-HP}^{(\mathrm{AutoReg})}(\omega)$ is the RC high-pass transfer function with cutoff frequency $f_c^{(\mathrm{AR})}$ that describes the effect of autoregulation.

Matlab code

The matlab code considers the following phasor ratios, which yield CHS spectra of amplitude ratios and phase differences:

$$\frac{\mathbf{D}(\omega)}{\mathbf{O}(\omega)} = \frac{|\mathbf{D}(\omega)|}{|\mathbf{O}(\omega)|}e^{i\{\mathrm{Arg}[\mathbf{D}(\omega)]-\mathrm{Arg}[\mathbf{O}(\omega)]\}}$$

$$\frac{\mathbf{O}(\omega)}{\mathbf{T}(\omega)} = \frac{|\mathbf{O}(\omega)|}{|\mathbf{T}(\omega)|}e^{i\{\mathrm{Arg}[\mathbf{O}(\omega)]-\mathrm{Arg}[\mathbf{T}(\omega)]\}}$$

Typical CHS spectra are as follows: