This paper outlines an approach to estimating the horizontal force-displacement response of well-confined reinforced concrete bridge piers. Particular emphasis is given to the hollow rectangular piers designed to support three new toll bridges in the San Francisco Bay Area. This approach accurately assesses a pier’s elastic displacement, its spread of plasticity and plastic displacement, and its shear displacement for most ductility levels. The elastic transfer mechanism inside a pier’s plastic hinge region is central to this assessment. This mechanism appears as a fanning crack pattern and results in concentrated compression strains at the base of a pier. These concentrated strains oppose the traditional notion of curvature that assumes that plane sections remain plane. The assumption that there is a linear distribution of plastic curvatures inside the plastic hinge region, however, largely overcomes the problem of relating plastic curvatures to plastic rotations, both experimentally and analytically. At nearly all ductility levels, the mean difference between analytical assessments of the spread of plasticity and results from 12 large-scale structural tests is 16% with a 12% coefficient of variation.

**Keywords:** bridges; plastic hinge; plasticity; reinforced concrete.

**INTRODUCTION**

In seismic regions, bridges are typically detailed according to capacity design principals, and the piers that support such bridges are designed as ductile members to deform inelastically under a large earthquake without losing strength. Furthermore, the displacement capacity of a bridge as a whole is often assumed to depend heavily on the displacement capacities of individual piers supporting the bridge. Moment-curvature analyses commonly form the basis for assessing the nonlinear force-displacement response of a particular reinforced concrete bridge pier. Such analyses require assumptions about the spread of plasticity in a structural member to calculate plastic rotations and displacements based on plastic curvatures. These analyses can be enhanced by accounting also for shear displacements and for fixed-end rotations resulting from strain penetration into the footing or bentcap.

One method for assessing plastic rotation that has strongly influenced ductile seismic design is based on the concept of the equivalent plastic hinge length, or “plastic hinge length.” This method assumes a given plastic curvature to be lumped in the center of the equivalent plastic hinge. The plastic hinge length is the length over which this plastic curvature, if assumed constant, is integrated to solve for the total plastic rotation. In addition to the plastic hinge length \( L_p \), this paper refers to the length of the plastic hinge region, \( L_{pr} \). The length of the plastic hinge region is the physical length over which plasticity actually spreads. The plastic hinge region \( L_{pr} \) refers to length along the pier only and, therefore, does not account for any penetration of inelastic strains into the footing or bentcap.

It is logical to expect that while \( L_p \) is not equivalent to \( L_{pr} \), \( L_p \) should be proportional to \( L_{pr} \). Such proportionality is not necessary, however, as long as inaccurate values of \( \phi_p \) and \( L_p \) combine to form an accurate value of \( \theta_p = \phi_p L_p \) that can be determined easily from experiments. The accuracy of \( \phi_p \) and \( L_p \) as separate components has, therefore, generated primarily academic discussion, where researchers have identified the effects of three distinct phenomena on the spread of plasticity in reinforced concrete members: moment gradient, tension shift, and strain penetration. Although each of those individual effects is well-established, there has been little resolution on the appropriate combination of these effects for assessing the spread of plasticity. Refer to Reference 5 for a detailed discussion of this history.

The term “moment gradient” reflects the fact that the transition between yield moment and ultimate moment in a member is proportional to the member’s shear span. Therefore, a member with a shorter shear span has a lower spread of plasticity. One accounts for this effect by assuming nonlinear material behavior in a structural member and also by assuming that plane sections before bending remain plane after bending. Applying these assumptions, one can assess the spread of plasticity based solely on the length of the member and the nonlinear moment-curvature relationship of a single section.

The term “tension shift” refers to the tendency of flexural tensile forces to decrease only minimally over a certain distance above the base of a pier until these forces can be transferred to the compression zone by adequately inclined struts. Tension shift invalidates the assumption that plane sections remain plane and allows plasticity to spread further than it would due to the effect of moment gradient alone.

The term “strain penetration” refers to the fact that longitudinal bar strains can reach significant inelastic levels some distance into the footing or bentcap. These strains taper to zero over a length required to develop sufficient bond strength for anchoring the bars against ultimate tensile forces. The accumulation of such strains inside the footing allows the flexural tension zone at the base of a pier to lift off the footing. This results in a fixed end rotation at the base of the pier.

Widespread use of the plastic hinge length has drawn heavily on an equation developed to model the behavior of simple circular and rectangular reinforced concrete bridge piers. This equation explicitly takes into account moment gradient and strain penetration effects and is written as...
Fig. 1—Schematic representation of proposed Bay Area bridge piers.

\[ L_p = 0.08L + 0.022d_p f_y \leq 0.044d_p f_y \text{ MPa} \]
\[ = 0.08L + 0.15d_p f_y \leq 0.30d_p f_y \text{ ksi} \]  

where \( L = M/V \) = member shear span; \( d_p \) = longitudinal bar diameter; and \( f_y \) = longitudinal bar yield stress. The two parameters \( d_p \) and \( f_y \) are combined to describe the strain penetration length \( L_p \). Equation (1) is typically applied in conjunction with ultimate curvatures that are limited according to the lower value from the equations

\[ \epsilon_u \leq \frac{\epsilon_{cc}}{c} \]  

and

\[ \epsilon_u \leq \frac{\epsilon_b + \epsilon_c}{D} = \frac{\epsilon_{cc}}{D'} \]  

where \( c \) = distance from the neutral axis to the extreme confined concrete compression fiber; \( \epsilon_{cc} \) = extreme confined concrete fiber strain from a moment-curvature analysis; \( \epsilon_b \) = extreme steel fiber strain; \( D' \) = distance between \( \epsilon_b \) and \( \epsilon_{cc} \); and \( \epsilon_{cc} \) = strain of the longitudinal reinforcement at ultimate stress. See Reference 1 for a detailed discussion of Eq. (1) to (3).

Equation (1) has no tension shift component because the data it was based on implied that the effects of tension shift were statistically insignificant. These data from the 1970s and 1980s were collected primarily from large-scale structural tests that modeled solid circular and rectangular bridge piers. While equations including the combined effects of moment gradient and tension shift on the spread of plasticity in reinforced concrete members have been proposed, they have received little attention in the area of bridge design. More recently, researchers have attempted to incorporate tension shift and its dependence on transverse reinforcement into the assessment of the plastic hinge length of bridge piers. This paper attempts to do the same, yielding results that apply more accurately to a wider range of cases.

Experimental results from large-scale tests based on these bridge piers have distinguished all three components of the plastic hinge length: moment gradient, tension shift, and strain penetration. Furthermore, attempts to calculate the experimental plastic hinge length for each test have brought the experimental plastic hinge length and the base curvature. Finally, the paper discusses the dependence of shear displacements on the kinematic behavior of the shear transfer mechanism in the plastic hinge region, and proposes a simple approach to estimating shear displacements in bridge piers.
RESEARCH SIGNIFICANCE

Moment-curvature-based approaches for assessing the horizontal force-displacement behavior of bridge piers typically aim to provide conservative numerical limits to the force and displacement capacity of standard bridge piers with solid circular and rectangular cross sections. Such approaches generally satisfy the minimum requirement of collapse prevention and have been applied successfully for many years to the seismic design and retrofit of bridges.

This paper outlines an approach to assessing the actual physical behavior of bridge piers at all levels of horizontal force and displacement in greater detail and with greater accuracy. This approach has proven effective for highly complicated hollow bridge piers as well as for simple solid bridge piers.

PROBLEMS IN ASSESSING EXPERIMENTAL SPREAD OF PLASTICITY

If tension shift can be described purely in terms of spreading plasticity, then it can be accounted for with a fair amount of rigor. The action of tension shift is, however, intimately linked to a concentration of compression strains at the section of maximum moment (this section is assumed to be at the base of a column). This concentration of compression strains complicates the notion of base curvature and increases the difficulty in calculating flexural deformations according to actual strain levels at the column base. Figure 3 compares an idealized crack pattern in the plastic hinge region of Test 3A, at a displacement ductility \( \mu_\Delta = (\Delta / \Delta_y) \) of 4, with experimental tensile and compressive strain distributions along the column height.

In addition to the concentration of compression strains, three more phenomena complicate the assessment of compression strain demand and capacity at the base of a column. The first is the inaccuracy inherent in the assumption of plane sections. Even if a section cuts through a column according to an idealized crack pattern, as Section A-A is shown to do in Fig. 3, there is no guarantee that the resulting compressive strain at the column base will correspond in magnitude to the compression strains derived from a “plane sections” moment-curvature analysis. Second, confinement provided to
Table 2—Test unit material properties and reinforcement schemes

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<td>683</td>
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</tbody>
</table>

Fig. 4—Test 3A, 12 curvature at \( \mu_A = 4 \).

where \( \Delta_p = \) elongation of a linear potentiometer on the tension side of the member (a positive number), and \( \Delta_c = \) shortening of a linear potentiometer on the compression side of the member at the same horizontal level (a negative number). Both potentiometers have the same gage length \( L_c \). Furthermore, \( D_\phi = \) distance between the two linear potentiometers, and \( \varepsilon'_p \) and \( \varepsilon'_c \) are experimental steel and concrete strains calculated from linear potentiometer readings. Figure 3 shows the linear potentiometers required by Eq. (4). The experimental strains shown in Fig. 3 are for the push direction only. Figure 4 shows the curvature distributions derived according to Eq. (4) for both the push and pull directions. Both Fig. 3 and 4 show more than one value of flexural strain or curvature at the column base. Two of these alternative measurements are labeled according to the subscript 0 for the case where \( L_g = \) the physical gage length at the base of the column, and the subscript \( sp \) for the case where \( L_g = L_g0 + L_{gpr} \) with \( L_{gsp} \) calculated according to Eq. (1). The third curvature value \( \phi_{sp} \) is called the effective base curvature and is explained as follows.

Although tension shift complicates the relationship between longitudinal strains and column deformation, the spread of plasticity can be modeled accurately based on the assumption that plastic curvatures are distributed linearly over the length of the plastic hinge region \( L_{gpr} \). Figure 3 shows that if strain penetration is accounted for in the calculation of the experimental longitudinal tensile strains at the column base \( (\varepsilon_{sh})_p \), the magnitude of tensile strains near the column base remains relatively constant. Because the compression strains in this same region increase to very high values near the column base, curvatures in this region maintain an approximately constant slope toward the column base, as shown in Fig. 4.

In 1994, Priestley, Seible, and Benzoni\(^{16}\) proposed that the experimental plastic hinge length could be assessed by assuming that the plastic curvatures were linearly distributed along a certain portion of the column height. Further observation of experimental data has shown consistently that plastic curvatures follow an approximately linear distribution inside the plastic hinge region\(^2\),\(^{10,13}\) (refer to Fig. 5 to 8). This linearity is disturbed only at the column base by the presence of both compressive and tensile strain penetration into the footing. In Fig. 4, the base curvatures \( \phi_{bsp} \) calculated according to Eq. (4) and (1) assuming strain penetration, have values that are similar to the base curvatures \( \phi_{sp} \) projected as least squares lines from plastic curvature values higher up in the plastic hinge region. This similarity implies that such a linear distribution method accurately approximates the physical behavior of the plastic hinge region by filtering out the fixed-end rotation attributed to compressive and tensile strain penetration.

ARGUMENT FOR CURVATURE

The experimental curvatures reported in this paper are calculated as

\[
\phi = \frac{\Delta_p - \Delta_c}{D_\phi L_g} = \frac{\varepsilon'_p - \varepsilon'_c}{D'}
\]

(4)
Assuming that plastic curvature is linearly distributed from the column base up to a height of \( L_{pr} \), and assuming that plastic rotation occurs primarily about the column base, \( L_p \) can be evaluated as

\[
L_p = \frac{\Delta_{pf}}{\phi_{p} L} = \frac{L_{pr}}{2} + L_{sp}
\]

(5)

where \( \Delta_{pf} \) = plastic flexural displacement. Refer to References 5 and 10 for the justification of Eq. (5) in contrast to previous formulations of \( \Delta_p \) as a function of \( \phi_p \) and \( L_p \).

To calculate experimental plastic curvature, the plastic flexural displacement is expressed as

\[
\Delta_{pf} = \Delta - \Delta_s - \Delta_{yf}' \frac{M}{M_y'}
\]

(6)

where \( \Delta = \) total experimental displacement; \( \Delta_s = \) experimental shear displacement; \( \Delta_{yf}' = \) experimental flexural displacement at first yield; and \( M = \) moment at the column base calculated as \( M = VL \), where \( V = \) measured actuator lateral force, and \( M_y' = \) theoretical first yield moment. The plastic curvature is expressed as

\[
\phi_p = \phi_y' - \phi_s \frac{M}{M_y'}
\]

(7)

where \( \phi_y' = \) theoretical first yield curvature. Refer to Reference 10 for a detailed discussion of the application of these experimental methods to the test units discussed in this paper.

**ASSESSING SPREAD OF PLASTICITY ANALYTICALLY**

The following section derives equations that characterize the spread of plasticity in bridge piers with axial load ratios \( P/L_{fc} \leq 0.20 \). These equations are based on parameters that can be characterized from moment-curvature analyses. In this study, the moment-curvature analyses accounted for nonlinear material behavior of longitudinal reinforcing steel, unconfined and confined concrete in compression according to Priestley, Seible, and Calvi\(^1\) and Mander, Priestley, and Park\(^17\) and tension stiffening in the unconfined and confined concrete according to Collins and Mitchell\(^18\).
Assuming that these parameters are available as output from a moment-curvature analysis for any level of curvature, the moment of resistance at the base of the column can be calculated exactly as

\[ M = VL = Tjd + Pd_P \]  

(8)

Figure 9(b) shows further assumptions made for the following derivations. The angled portion of the crack at Section A-A is inclined at an angle \( \theta \) from the vertical and extends straight from the compression resultant \( C \) to the flexural tension resultant \( T_1 \), reaching a height of \( L_{aj} = jdcot\theta \). The tensile stress carried by the transverse reinforcement \( \sigma_s \) is assumed to be concentrated at a height of \( jdcot\theta/2 \) as the resultant horizontal force \( V_{cr} \). Furthermore, the horizontal tensile stresses in the concrete are assumed to act as the resultant horizontal force \( V_{cr} \) at a distance \( d_{cr} \cot\theta \) above the base of the column.

The value

\[ T_{yav} = \frac{T_1 + T_2}{2} \]  

(9)

is applied as the effective flexural tensile yield force resultant, where \( T_1 \) = flexural tensile force resultant at first yield of the extreme steel fiber, and \( T_2 \) = flexural tensile force resultant when either \( \varepsilon_c = 0.004 \) or \( \varepsilon_c = 0.015 \) is first reached \(^1\) (\( M_y \) and \( M_y' \) are also calculated at these levels). This yield force includes the tensile forces produced by both the reinforcing steel and the concrete. Equations for \( L_{avg} \) are sensitive to this effective yield value, and Eq. (9) appears to estimate this value accurately. Other methods for assessing this value, such as determining it according to the same level of curvature ductility for all test units, or assessing it according to the values \( \phi_y \) or \( \phi_y' \) \( (M_y/M_y') \), were investigated but found inferior to Eq. (9). Figure 10 shows a plot of the relationship between \( T \) and \( L_{avg} \) for Test Unit 2C, where \( T_{yav} \) best captures the transition between preyield and postyield behavior of the flexural tensile force resultant.

Assuming that \( T_{yav} = T_{yav} \) and that the stresses on Section A-A in Fig. 9(a) can be approximated by the resultant forces in Fig. 9(b), moment equilibrium gives

\[ (T - T_{yav})jd - (V_s + V_{cr} \frac{2d_{cr}}{jd}) \frac{jdcot\theta}{2} = 0 \]  

(10)

Assuming that all transverse steel in the plastic hinge region has yielded, \( V_s \) can be expressed as

\[ V_s = \frac{A_y f_y}{d_{cr} \cot\theta} \]  

(11)

where \( A_y \) = area of transverse steel at a given level; \( f_y \) = yield stress of the transverse steel; and \( s \) = spacing between the transverse bars. Table 2 gives these values from all 12 test units.

The horizontal concrete tensile force resultant \( V_{cr} \) is estimated based on values derived from moment-curvature analysis as explained as follows. The average flexural tensile concrete stress is estimated as

\[ t_{cr} = \frac{V_{cr}}{(D - c)t_w} \]  

(12)
where \( t_w \) = effective width of the section. This vertical, flexural tensile concrete stress is assumed to relate to horizontal and principal stresses in the concrete as

\[
t_{cr} = v_{cr} \tan \theta = f_j \sin \theta
\]  

This assumption is inconsistent with the general approach presented herein. Nevertheless, detailed calculations of both the concrete and steel horizontal forces according to the presented herein. This assumption is inconsistent with the general approach presented herein. Nevertheless, detailed calculations of both the concrete and steel horizontal forces according to the nonlinear Modified Compression Field Theory\(^{18}\) proved that Eq. (14) did not affect the predicted spread of plasticity any more than the assumption of uniform yielding in the transverse steel in Eq. (11). The resultant flexural tensile concrete force \( T_{cr} \) and its resultant location \( d_{cr} \) were calculated at the same curvature level as \( T_{yar} \) from Eq. (9). Furthermore, these effective yield values were not assumed to vary between Section A-A and B-B in Fig. 9.

Based on Eq. (12) and (14), \( V_{cr} \) is approximated in a form similar to Eq. (11).

\[
V_{cr} = \frac{v_{cr} t_w j d \cot \theta}{1.4}
\]  

where \( v_{cr} t_w \) = average distributed horizontal tensile force in the concrete.

Combining Eq. (10), (11), and (15) results in the equation

\[
L_{pr} = \frac{j d \cot \theta}{1.4} = \frac{2(T - T_{yar}) j d}{s (A_f \frac{f_{yy}}{s} + v_{cr} t_w \frac{2 d_{cr}}{j d})}
\]  

While Eq. (16) describes the effect of tension shift on the spread of plasticity, it only partly describes the effect of moment gradient. The expression \( T - T_{yar} \) captures the difference between the yield moment and values above the yield moment, but it does not capture the possible spread of plasticity into a shear span’s uniform diagonal stress field. This spread is dependent on the length of the shear span \( L \). In a slender, well-confined column such as TU1,\(^{14}\) plasticity tends to spread much further than the spread indicated by Eq. (16). This is shown clearly by the plastic curvature profiles for TU1 presented in Fig. 5. For Eq. (16) to predict such spread of plasticity the crack angle marking the end of the stress field in the plastic hinge region would have to reach \( \theta < 15 \) degrees.

In response to such cases, the crack angle should be limited by the estimated shear cracking behavior of the column. Plasticity is assumed to spread according to Eq. (16) only until the inclination of compression struts corresponds to \( \cot \theta_1 \), where \( \theta_1 \) = crack angle limited according to shear and not according to spreading plasticity. The flexural tensile force resultant at this level \( T_1 \), as shown in Fig. 9, is therefore no longer equivalent to the effective tensile yield force \( T_{yar} \).

Based on Eq. (10), (11), and (15), \( T_1 \) can be estimated as

\[
T_1 = \frac{V_{cr} t_w j d}{1.4} + \frac{2 d_{cr}}{j d}
\]  

and the diagonal crack angle \( \theta_1 \) can be estimated according to Collins and Mitchell\(^{18}\) as

\[
\theta_1 = \cot^{-1} \left( \frac{V}{\left( \frac{A_f \frac{f_{yy}}{s}}{s} + f_j \frac{t_w}{j d} \right)} \right) < 90 \text{ degrees}
\]  

where the principal tensile concrete stress \( f_j = 1.4 t_{cr} \) (refer to Eq. (14)). Combining this assumption with the observation that \( (2d_{cr} / j d) \approx 1.4 \) for all 12 test units allows Eq. (17) and (18) to be combined very simply as

\[
T_1 = T - \frac{V \cot \theta_1}{2} \geq T_{yar}
\]

Considering the free-body diagram between Sections A-A and B-B in Fig. 9, where relevant equilibrating vertical and horizontal forces on the top portion are shown as dashed arrows, \( L_{mg} \) can be solved based on moment equilibrium as

\[
L_{mg} = \left( T_{1} - T_{yar} \right) \frac{j d}{V} \geq 0
\]

Further combining Eq. (18) to (20) allows for \( L_{pr} \), to be calculated as

\[
L_{pr} = \left( T - T_{yar} \right) \frac{j d}{V} + \frac{V}{2 \left( \frac{A_f \frac{f_{yy}}{s}}{s} + f_j \frac{t_w}{j d} \right)} \geq 0
\]

### COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

Figure 5 to 8 compare Eq. (21) (shear crack model) and Eq. (16) (bond stress model) to some of the experimental results, showing that both equations adequately predict the spread of plasticity at most levels of \( \mu_h \). Both the experiment and the theories show plasticity spreading further with increasing curvature ductility. They also show plasticity spreading rapidly between \( 11 = 1 \) and \( \mu_h = 2 \) and then slowing down for higher levels of displacement ductility. (Note that while all experimental and analytical data are reported according to curvature ductility \( \mu_h \), experimental data was collected at levels of displacement ductility \( \mu_h \) specified in round numbers.) For well-confined, slender, circular columns such as Test TU1,\(^{14}\) Eq. (21) provides more accurate results than Eq. (16) (refer to Fig. 5). For slender structural walls with boundary elements such as Tests 1A and 1B, however, Eq. (16) provides more accurate results than Eq. (21) (refer to Fig. 6). For structural walls with boundary elements that have lower aspect ratios, Eq. (21) and (16) predict the spread of plasticity with comparable accuracy (refer to Fig. 7 and 8).

Table 3 compares these two models to experimental results for values of \( L_{pr} \), \( L_{mg} \), and \( \Delta \). Column (1) identifies the test. Columns (2) to (13) report the mean differences between analysis and experiment and their coefficients of variation.
Table 3—Mean differences and coefficients of variation for analytical and experimental values of $L_{pr}$, $L_p$, and $\Delta$

<table>
<thead>
<tr>
<th>Test (1)</th>
<th>$\Delta_{Lp, mm}$</th>
<th>$\Delta'_Lp, mm$</th>
<th>$\Delta_{exp, mm}$</th>
<th>$\Delta_{Lp}'$, %</th>
<th>$\Delta_{exp}'$, %</th>
<th>$\phi_{02}$, mm</th>
<th>$\phi_{06}$, mm</th>
<th>$\Delta_02, mm$</th>
<th>$\Delta_06, mm$</th>
<th>$\Delta_{exp, mm}$</th>
<th>$\Sigma_{fail}$, %</th>
<th>$\Sigma_{u}$, %</th>
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<tr>
<td>TU1</td>
<td>31.5</td>
<td>31.5</td>
<td>24.2</td>
<td>30.4</td>
<td>30.4</td>
<td>91.8</td>
<td>262</td>
<td>$\epsilon_u$</td>
<td>85.9</td>
<td>162</td>
<td>320</td>
<td>$-$18.3</td>
</tr>
<tr>
<td>C3</td>
<td>25.9</td>
<td>25.9</td>
<td>22.2</td>
<td>16.6</td>
<td>16.6</td>
<td>92.2</td>
<td>162</td>
<td>$\epsilon_u$</td>
<td>56.4</td>
<td>138</td>
<td>17.34</td>
<td>$-$47.8</td>
</tr>
<tr>
<td>1A</td>
<td>25.2</td>
<td>23.4</td>
<td>21.7</td>
<td>16.4</td>
<td>7.85</td>
<td>42.9</td>
<td>218</td>
<td>$\epsilon_u + \epsilon_s$</td>
<td>103</td>
<td>277</td>
<td>226</td>
<td>$-$3.27</td>
</tr>
<tr>
<td>1B</td>
<td>25.4</td>
<td>23.1</td>
<td>21.3</td>
<td>19.0</td>
<td>8.21</td>
<td>42.9</td>
<td>248</td>
<td>$\epsilon_u + \epsilon_s$</td>
<td>103</td>
<td>277</td>
<td>226</td>
<td>10.02</td>
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<tr>
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<td>5.72</td>
<td>6.35</td>
<td>5.60</td>
<td>$-$10.0</td>
<td>42.9</td>
<td>81.3</td>
<td>$\epsilon_u + \epsilon_s$</td>
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<td>80.8</td>
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<td>5.89</td>
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<td>26.3</td>
<td>74.2</td>
<td>$V &gt; V_u$</td>
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<td>20.4</td>
<td>130</td>
<td>$V &gt; V_w$</td>
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<td>$-$31.0</td>
<td>43.7</td>
<td>264</td>
<td>$\epsilon_u$</td>
<td>93.0</td>
<td>266</td>
<td>276</td>
<td>$-$4.24</td>
</tr>
<tr>
<td>DPT(LU)</td>
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<td>32.8</td>
<td>$-$0.93</td>
<td>$-$29.5</td>
<td>43.7</td>
<td>277</td>
<td>$\epsilon_u$</td>
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<td>262</td>
<td>295</td>
<td>$-$6.03</td>
</tr>
<tr>
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<td>$\epsilon_u + \epsilon_s$</td>
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<td>25.8</td>
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<td>15.5</td>
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</tbody>
</table>

Previous research has shown that equations such as Eq. (16) and (21) can be effectively paired with base curvatures that are limited according to the relationship

$$\phi_u \leq \frac{|\epsilon_c| + |\epsilon_s|}{D'} = \frac{0.05}{D'}$$  \hspace{1cm} (22)

This relationship accounts empirically for the effects of longitudinal bar buckling, which is believed to be the ultimate cause of failure in confined concrete. Ultimate curvatures calculated according to Eq. (22) are listed as $\phi_{02}$ in Column (7) of Table 4. Ultimate curvatures calculated according to Eq. (2) and (3) are listed as $\phi_{06}$ in Column (10) of Table 4, where it is assumed that $\epsilon_{u2} = 0.12$.

Based on moment-curvature analysis, Eq. (16), (21), and (22), and equations for elastic displacement and shear

(\text{COV}) for the values $L_{pr}$ (Eq. (21) and (16)), $L_p$ (Eq. (5)), and $\Delta$ (Eq. (23)). Mean differences between experiment and analysis are reported for all ductility levels ranging from $\mu_{\Delta} = 2$ to failure of the test units.

The mean difference for all of the tests combined is 16%, with a 12% COV (Table 3, Eq. (21)). Three test units in particular were responsible for keeping the models from proving more accurate. These test units were C3,15 the lightly confined circular column; 2B,11 the structural wall with a low aspect ratio and very light transverse reinforcement; and the bottom portion of the DPT;13 the hollow pier that was subjected to biaxial bending. Excluding these three test units, the average difference between the theoretical spread of plasticity $L_{pr}$ and the experimental behavior at all ductility levels was 13% with a 10% COV.
displacement introduced in later sections, the displacement can be calculated for all inelastic levels of curvature ductility as

\[
\Delta = \Delta_\gamma' + \left(\phi - \phi_\gamma'\right)M_y \left[1 + \Delta_\gamma'\Delta_y'\right] = \Delta_u(\phi_u) \tag{23}
\]

Before studying Table 4, which compares the experimental and analytical ultimate displacements, consider that test unit failure at \(\mu_\Delta = 8\) implies 33% more displacement capacity than failure at \(\mu_\Delta = 6\). Therefore, it is difficult to expect even experimental ultimate displacements to be more accurate than this discrepancy. Failure at a given level of displacement ductility in some ways depends as much on the definition of the loading history as it does on the actual behavior of the column.

Equation (1) modeled \(L_{sp}\) with a level of accuracy that allowed the value of \(L_p\) to differ from experimental values of \(L_p\) with a mean difference of 16% and a 13% COV. Table 3 shows this level of error to correspond almost exactly to the level of error calculated for the value \(L_{sp}\) from Eq. (21). Some variations in the experimental value of \(L_{sp}\) resulted from varying levels of reinforcement and post-tensioning in different footings.

Table 4 compares ultimate displacements, \(\Delta_{u2}\) in Column (8), based on Eq. (21) to (23) to the ultimate displacements, \(\Delta_{u96}\) in Column (11), calculated according to Reference 1. The subscript 02 refers to the fact that the equations were developed in 2002, whereas the subscript 96 refers to the equations published by Priestley, Seible, and Calvi in 1996 as the state of the art in seismic bridge design. It is not possible to compare these two approaches except at ultimate displacements because they rest on dissimilar notions of base curvature. Column (9) gives the reason for terminating each moment-curvature analysis. In the cases of Tests 2B, 3A, 3B, and 3C, the failure criteria related to shear failures at a given level of curvature ductility. Refer to Reference 5 for a detailed discussion of these shear failures. The flexural failure criteria for \(\Delta_{u96}\) values from other tests are labeled either \(\varepsilon_{cy}\) denoting limits according to Eq. (2), or \(\varepsilon_c + \varepsilon_y\), denoting limits according to Eq. (3). For the \(\Delta_{u2}\) values, in Column (8), all flexural failure limits were determined according to Eq. (22). Figure 11 to 13 show the full hysteretic behavior and force-displacement characterizations for Test Units TU1, 1A, and 2A. Each of these three test units failed in flexure by buckling and fracture of the longitudinal reinforcement at \(\mu_\Delta = 8\).

\[\Delta y' \phi y' L y' \frac{2}{\phi y'} = \Delta y' \text{ (24)}\]

Fig. 11—Test Unit TU1. Hysteretic behavior with force-displacement characterizations.

Fig. 12—Test Unit 1A. Hysteretic behavior with force-displacement characterizations.

Fig. 13—Test Unit 2A. Hysteretic behavior with force-displacement characterizations.

Values of \(\Delta_{u96}\) in Column (11) are indeed conservative by roughly 50% for the circular columns (as Priestley, Seible, and Calvi stated in 1996) when compared with the experimental ultimate displacement values, \(\Delta_{uexp}\) in Column (12). These values, however, slightly over-predict values of ultimate displacements for the hollow piers and structural walls that failed in flexure. While the accuracy of these equations with regard to the ultimate displacement capacity of structural walls is noteworthy, it raises the question of whether they are reliably conservative by 50%. Furthermore, their conservatism for walls that failed in shear appears to be accidental. In summary, Eq. (16) and (21) to (23) are distinguished as much for their ability to correlate curvatures and displacements at most levels of curvature ductility (Table 3) as for their ability to assess ultimate displacement capacity accurately (Table 4).

**PREYIELD STRAIN PENETRATION**

The theoretical displacement at first yield assessed as

\[\Delta_y' = \frac{\phi_y' L y'^2}{3}\]
Tests 3A, 3B, 3C LPT, DPT(L), and DPT(T). Table 4 displays all of these values, where \( \Delta_{y6} \) in Column (3), corresponds to Eq. (24), and \( \Delta_{\text{spy}} \) in Column (4), corresponds to the experimental first yield displacements. The experimental first yield displacements measured from the tests when they were pushed and pulled in load control through one full cycle to \( F'_{y} \) = theoretical force corresponding to first yield of the longitudinal reinforcement as assessed according to moment-curvature analysis.

Overprediction of \( \Delta'_{y} \) for Tests TU1 and C3 most likely resulted from the assumption that elastic curvatures are linearly distributed from the column base to the point of contraflexure. This assumption does not account for the fact that elastic curvature as TU1 and C3, which had aspect ratios of \( M/VD = 6 \), a significant portion of the columns remained uncracked prior to yield and during the entire course of testing. Tall structural walls such as 1A and 1B, which have aspect ratios of \( M/VD \approx 4 \), also experienced lower experimental first-yield displacements than those predicted by Eq. (24).

Underprediction of Tests 3A, 3B, 3C, LPT, and DPT resulted from neglecting preyield strain penetration and elastic shear displacements. Preyield strain penetration in Test 1A, 1B, 2A, and 2C was not significant because the footings of these test units had been laterally post-tensioned. While preyield strain penetration has been recognized by researchers at least during the past decade,\(^{19} \) it has not been accepted widely. Furthermore, preyield strain penetration has been discussed as a phenomenon that results primarily from the tensile straining of longitudinal bars into the footing. Figure 14 shows, however, that strain penetration also occurs in the compression region prior to yield. Figure 14 also shows that phenomena such as tension shift and compression concentration typically associated with plastic hinging also occur prior to yield. Comparing Test Units 3A, 3B, and 3C in Fig. 14, preyield strain shift and concentration of compression at the base of the column increase with increasing section depth.

Figure 14 shows the strain values near the column base as recorded by Test 3A strain gages SL2A and SL2G.\(^{12} \) If \( \Delta_{2A} \) is calculated by integrating the strains inside the footing along Bar 2A, \( \Delta_{2G} \) is calculated by integrating the strains inside the footing along Bar 2G, and the curvature at the column base, \( \phi_{p} \), is calculated by moment-curvature analysis for the given load level, then the preyield strain penetration length can be calculated from the integrated displacements and the theoretical base curvature as

\[
L_{\text{spy}} = \frac{\Delta_{2A} - \Delta_{2G}}{D' \phi_{p}} \quad (25)
\]

For Tests 3A, 3B, 3C, LPT, and DPT, the mean value of \( L_{\text{spy}} \) for all load levels was \( L_{\text{spy}} = 17.1d_{p} \), implying that \( L_{\text{spy}} = 2L_{sp} \), where \( L_{sp} \) is calculated according to Eq. (1). Table 4 gives theoretical first yield displacement values, \( \Delta'_{y02} \) in Column (2), calculated as

\[
\Delta'_{y02} = \phi_{p}'L_{3} + L_{\text{spy}}L_{2} \left( 1 + \frac{\Delta_{y}}{\Delta'_{y02}} \right) \quad (26)
\]

where the ratio \( \Delta_{y}/\Delta_{y02} \) accounts for shear displacements and is explained in the following section. Tests TU1 and C3 are calculated according to Eq. (24); Tests 1A, 1B, 2A, 2B, and 2C are calculated according to Eq. (26), assuming that \( L_{\text{spy}} = 0 \) (because of the lateral post-tensioning in the footings); and Tests 3A, 3B, 3C, LPT, and DPT are calculated according to Eq. (26) assuming that \( L_{\text{spy}} = 2L_{sp} \). Equation (26) demonstrates the greater effect that preyield strain penetration has on shorter columns. The closer the expression \( L_{\text{spy}}/L \) comes in magnitude to the ratio \( L/3 \), the greater effect preyield strain penetration will have on elastic displacements.

**SHEAR DISPLACEMENTS**

For the hollow pier and structural wall test units, with aspect ratios of \( M/VD \leq 4 \), shear displacements ranging from 7 to 28% of the total measured displacement were commonly observed. As expected, the contribution of shear displacements increased with decreasing aspect ratio in all test units.

Dazio has observed that shear displacements are often linearly related to flexural displacements through the entire range of displacement capacity.\(^{20} \) This linear relationship, at least between peak displacements, was confirmed for 10 of the tests discussed in this paper.\(^{5} \)

Because the largest portion of shear displacements was commonly observed inside the plastic hinge region, a simple formula for shear displacements in the plastic hinge region can be derived based on the kinematics of the fanning crack pattern and Dazio’s observation. This derivation is given in the Appendix to this paper. It results in the equation

\footnote{The Appendix is available in xerographic or similar form from ACI headquarters, where it will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.}
\[ \frac{\Delta_s}{\Delta_f} = \alpha \frac{0.35(1.6 - 0.02\theta_m)}{L} \]  

(27)

where \(\theta_m\) = crack angle measured in from the vertical in degrees at the level of maximum displacement, and where

\[ 1 \leq \alpha = \left( \frac{V_f}{V_{nf}} + \frac{V_{swf}}{V_{wcf}} \right) \leq 2 \]  

(28)

which accounts for cracking inside of the plastic hinge region but outside of the fanning crack pattern. In Eq. (28), \(V_{nf}\) = the member’s shear capacity under diagonal tension, and \(V_{wcf}\) = the member’s shear capacity under diagonal compression. Refer to Reference 5 for a detailed explanation of both these terms.

When used with Eq. (23) to calculate ultimate displacement, Eq. (27) gives analytical values that differ from experimental values with a mean of 4.4% and a COV of 4.4% for the 10 test units studied (which do not include the two circular columns). Evaluated on its own, Eq. (27) results in analytical values that differ from experimental values with a mean of 21% and a COV of 18%. The difference most likely results from the crudeness of Eq. (28).

**SUMMARY AND CONCLUSIONS**

A comparison of moment-curvature-based force-displacement characterizations with results from 12 large-scale cyclic tests demonstrated the possibility of assessing actual force-displacement mechanisms of well-confined bridge piers at most ductility levels. Only displacement levels at the very beginning of the plastic range, around \(\mu = 1\), proved to be difficult to model. This is explained by the choice of a specific flexural tensile yield force \(T_{yun}\). This force determines both the level of curvature ductility where plasticity begins to spread and its initial rate of spread. Slight variations in this force will cause proportionally large variations in initial values of \(L_{pr}\) corresponding to demand levels around \(\mu = 1\).

Key concepts for relating curvature ductility to displacement ductility included the influence of the plastic hinge shear transfer mechanism on the spread of plasticity and shear displacements and the influence of compressive and tensile strain penetration on elastic displacements.

Primary conclusions are enumerated as follows:

1. Assuming a linear distribution of plastic curvatures allows for the spread of plasticity in a bridge pier to be defined in terms of moment gradient and tension shift effects. The approach given by Eq. (21) modeled the spread of plasticity to with a 16% mean difference and a 12% COV over all displacement ductility levels from \(\mu = 2\) to failure for all test units. This approach may be applied to a wide range of member geometries with axial load ratios of \(P/L_{App} < 0.20\), such as the piers supporting the new East Bay Skyway of the San Francisco-Oakland Bay Bridge. The authors have not thoroughly investigated the application of this approach to members with higher axial load ratios. If the plastic curvature distribution for a given column is assumed to not to have a linear distribution, Eq. (5) must be modified accordingly;

2. Strain penetration had a more pronounced effect on the elastic displacement than on plastic displacement because the strain penetration gave rise to a greater portion of the total displacement prior to yield. Test results showing compression strain penetration and elastic compression concentrations suggested that base rotations due to penetrating tensile strains only may account for only half of the total elastic strain penetration in some bridge piers;

3. If the elastic displacement can be predicted accurately for a structural wall or hollow rectangular pier with well-confined boundary elements \((\rho > 0.008)\) and adequate shear capacity, then the ultimate displacement can often be estimated as simply \(6 \times \Delta_t\) for the purposes of design, where \(\Delta_t = \Delta_t'(IMC)\) = ideal yield displacement. Test units failing in flexture experienced buckling and fracture of the longitudinal reinforcing bars some time between \(\mu = 6\) and \(\mu = 8\). Why this was true may be explained rigorously when more insight has been acquired into real strain limit states; and

4. The kinematics of the shear transfer mechanism provided insight into the expected magnitude of shear displacements in a bridge pier. This mechanism, the assumption that \(\Delta_t/\Delta_f\) is a constant, and the accurate assessment of shear capacity in diagonal tension and diagonal compression formed the basis for approximating inelastic shear displacements with a mean difference of 21% and an 18% COV. This resulted in a mean difference between theoretical and experimental values for overall displacements of 4.4% with a 4.4% COV. For the 10 test units from which shear displacements were calculated (refer to the Appendix), ranging between 0.07 \(\leq \Delta_t/\Delta_f \leq 0.038\), the member aspect ratio, the maximum crack angle in the plastic hinge region, and shear capacity proved to be the parameters necessary for evaluating \(\Delta_t/\Delta_f\).

While the force-displacement characterization has been evaluated for only 12 test units, it has proven the possibility and usefulness of assessing in detail the force-displacement behavior of bridge piers both experimentally and analytically. This level of detail is necessary for accurately assessing the force and displacement at all levels of curvature ductility. It also gives insight into design by assessing the physical consequences of specific design details.

**ACKNOWLEDGMENTS**

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**NOTATION**

- \(A_g\) = gross area of section
- \(A_{tr}\) = area of transverse steel at given level
- \(C\) = net compressive force
- \(c\) = neutral axis depth
- \(c_f\) = flexural compression stress
- \(D\) = member total section depth
- \(D'\) = distance between extreme fiber steel and confined concrete strains
- \(D_{sk}\) = distance between curvature potentiometers
- \(DPT(LL)\) = diagonal pier test (SFOBB East Bay Skyway) bridge longitudinal direction, lower hinge
- \(DPT(LL)\) = diagonal pier test, bridge longitudinal direction, upper hinge
- \(d_p\) = bar diameter

The Appendix is available in xerographic or similar form from ACI headquarters, where it will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.
\[d_{tr} = \text{distance between } T_r \text{ and } C, \text{ from moment-curvature analysis}\]
\[d_{p} = \text{distance between axial load and compression centroid}\]
\[E_{dh} = \text{reinforcing steel strain hardening modulus}\]
\[F_{y} = \text{first yield force}\]
\[f_{p} = \text{principal tensile stress}\]
\[f_{cx} = \text{unconfined concrete strength}\]
\[f_{c} = \text{ultimate steel stress}\]
\[f_{l} = \text{longitudinal reinforcement yield stress}\]
\[f_{sy} = \text{transverse reinforcement yield stress}\]
\[j_{d} = \text{distance between flexural tension and compression centroids}\]
\[L = \text{shear span} \left( L = \frac{M}{V} \right)\]
\[L_{g} = \text{gage length of linear potentiometer}\]
\[L_{mg} = \text{moment gradient portion of spread of plasticity}\]
\[L_{p} = \text{plastic hinge length}\]
\[L_{pr} = \text{plastic hinge region length}\]
\[L_{sp} = \text{strain penetration length}\]
\[L_{spy} = \text{preyield strain penetration length}\]
\[L_{t} = \text{tension shift portion of spread of plasticity}\]
\[M = \text{bending moment}\]
\[M_{e} = \text{ideal yield moment}\]
\[M_{f} = \text{first yield moment}\]
\[M_{VD} = \text{member aspect ratio}\]
\[P = \text{axial load; steel strain hardening exponent}\]
\[SFOBB = \text{San Francisco-Oakland Bay Bridge}\]
\[T = \text{flexural tensile force resultant}\]
\[T_{1} = \text{flexural tensile force resultant at height } j_{d} \cot \theta_{1}\]
\[T_{cr} = \text{vertical flexural tensile concrete force resultant}\]
\[T_{T} = \text{flexural tensile force resultant at ideal yield}\]
\[T_{t} = \text{flexural tensile force resultant at first yield}\]
\[T_{uy} = \text{effective flexural tensile yield force resultant}\]
\[t_{cr} = \text{flexural tensile concrete stress}\]
\[t_{i} = \text{vertical flexural tensile stress from longitudinal bars}\]
\[V = \text{horizontal force applied to test unit}\]
\[V_{cr} = \text{horizontal resistance provided by concrete tensile stresses}\]
\[V_{n} = \text{shear resistance to diagonal tension}\]
\[V_{h} = \text{horizontal resistance provided by transverse steel}\]
\[V_{wc} = \text{shear resistance to web crushing}\]
\[v_{cr} = \text{horizontal tensile stress}\]
\[v_{t} = \text{shear crack angle calculated according to spread of plasticity}\]
\[\phi = \text{curvature; strength reduction factor}\]
\[\phi_{b} = \text{base curvature}\]
\[\phi_{p} = \text{plastic curvature}\]
\[\phi_{u} = \text{ultimate curvature}\]
\[\phi_{06} = \text{ultimate curvature} \left( \text{Eq. (2) and (3)} \right)\]
\[\phi_{02} = \text{ideal yield curvature}\]
\[\phi_{f} = \text{first yield curvature}\]
\[\mu_{A} = \text{displacement ductility}\]
\[\mu_{f} = \text{curvature ductility}\]
\[\theta_{0} = \text{shear crack angle calculated according to spread of plasticity}\]
\[\theta_{1} = \text{shear crack angle calculated according to shear capacity}\]
\[\theta_{m} = \text{minimum shear crack angle used for shear displacements}\]
\[\rho_{f} = \text{plastic rotation}\]
\[\rho_{l} = \text{boundary element longitudinal reinforcement ratio}\]
\[\tau_{s} = \text{volumetric confinement ratio}\]

**REFERENCES**