IMPEDANCE MODELING FOR PREDICTION OF TRAIN INDUCED FLOOR VIBRATIONS

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ABSTRACT

Laboratory and manufacturing facilities are frequently located in areas that experience high-frequency (greater than 30 Hz) environmental vibration. Such vibrations are a concern for the proper functioning of sensitive laboratory devices, precision manufacturing equipment as well as human comfort. A simple method to accurately predict the magnitude of high-frequency floor vibrations would allow detailed vibration assessment during the conceptual design phase, and provide physical insight useful for developing vibration mitigation approaches. This paper discusses a methodology for prediction of train-induced floor vibrations in buildings using a wave propagation technique. The data obtained from an experiment carried out at Tufts University on a scale model laboratory building have been used to validate the predictions. The proposed wave propagation method successfully predicted the measured velocities and impedances of the scale model building.

INTRODUCTION

Many facilities are located in environments with substantial ambient vibration. The vibration may be from highway vehicles, trains, or other sources. Floor vibrations are of concern for precision manufacturing facilities and for laboratories where sensitive experiment is carried out. Examples of sensitive manufacturing processes include fabrication of laser devices, electronic microscopes, surgical devices, and semiconductor integrated cores. As technologies continue to evolve and become more complex, the demands on manufacturing are expected to become more rigorous. These vibrations also may be of concern for human comfort in residential settings.

This paper focuses on the experimental validation of a simple wave propagation based modeling approach for the prediction of train-induced floor vibrations. This approach can be used during the conceptual design phase of projects where owners
are concerned about environmentally induced floor vibrations—with the potential to save millions of dollars in construction costs and hundreds of thousands of dollars detailed finite element studies whose accuracy is not guaranteed in the high frequency range.

BACKGROUND

Problems associated with floor vibrations are not new. The first known study was reported almost 70 years ago (Hunaidi et al., 1997). Numerous studies have been carried out to contribute to the understanding of the problem of floor vibrations.

Bahrekazemi (2004) reported that the techniques used for vibration mitigation are location dependent. These locations are either at the vibration source, in the path, or at the structure. Ju et al. (2004), Adam et al. (2005), and Wang et al. (2003) applied these various mitigation methods for the reduction of train induced vibrations. Mitigation methods selected were based on the location including soil improvement under the embankment and construction of concrete slabs, construction of open as well as in-filled trenches and isolation of the building foundation from the ground using elastic support system. Site conditions also play a dominant role in deciding the most appropriate mitigation method.

For successful mitigation of floor vibrations, it is necessary to have accurate and efficient prediction methods. In order to predict building vibration induced by nearby trains, various approaches have been adopted. Some of these approaches include finite element modeling, numerical model based approach, utilization of transfer functions and impedance based models. Each of these methods is characterized by its own efficiency, accuracy, ease of use in application, time consumption and limitations.

Ju (2007) developed an integrated finite element model for a bridge, nearby building, soil, and train for vibration prediction. Gupta et al. (2008) developed a coupled periodic finite element and boundary element model. It was used for prediction and assessment of subway induced vibrations with respect to studying the performance of different track structures in tunnels. With et al., (2007) used transfer functions for the prediction of floor vibrations in a building. Trochides (1991) predicted excitation levels due to ground borne vibrations in buildings near subways based on the approximate impedance formulae for the tunnel and the structure and energy considerations.

In the present paper, an approach for floor vibration prediction based on impedance modeling has been used. The authors developed an analytical model based on their previous research published by Sanayei et al. (2008), Hughes et al. (2008), and Zhao et al. (2010) to predict the velocity response of the structure. Experiments using a shaker on a scale model building have been carried out to simulate train-induced vibrations. The measured floor velocities have been used to validate the wave propagation model predictions. This paper also explains the efficiency and applicability of this approach in predicting floor vibrations in buildings.
MATHEMATICAL MODEL FOR IMPEDANCE BASED APPROACH

Train-induced vibrations propagate through soil in the form of waves that are transmitted to buildings at the foundation level. The vibrations are predominantly vertical although horizontal components exist (Li et al. 2009; Yanmei et al. 2010). The interaction between the ground borne vibration and the vibration induced in the building foundation is complicated. In the present paper, it is assumed that the dominant mode of transmission into the upper floors in buildings is from longitudinal waves (dilatation) in the columns, bending wave transmission is more readily blocked at the lower floors. In the mathematical model, the vibrating force is assumed to be applied only in the vertical direction at the base of the columns (Conroy, 2008). Floor plates attached to the columns radiate energy as transverse bending waves. Longitudinal waves transmitted in columns are less affected by floors of typical weights and are more importantly responsible for the transmission into the upper floors.

**Modeling of Column.** The partial differential equation for free vibration of a column subjected to longitudinal waves (see Figure 1) is given by (deSilva, 2000)

\[
\rho A(x) \frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial}{\partial x} \left[ E_c A(x) \frac{\partial u(x,t)}{\partial x} \right] = 0
\]

where \( \rho \), \( A \) and \( E \) are density, cross-sectional area and modulus of elasticity, respectively. \( E_c \) is the complex modulus of elasticity representing energy dissipation as \( E_c = E(1 + j\eta) \) with \( \eta \) denotes the structural loss factor.

The steady state response is given by:

\[
u(x,t) = u(x)e^{j\omega t}
\]

By substituting (2) into (1), the spatial equation of motion thus obtained is,

\[
-\rho A(x)\omega^2 u(x) - \frac{\partial}{\partial x} \left[ E_c A(x) \frac{\partial u(x)}{\partial x} \right] = 0
\]

Furthermore, for a prismatic bar element, (3) can be written as,

\[
\frac{\partial^2 u(x)}{\partial x^2} + \beta^2 u(x) = 0
\]

where

\[
\beta = \omega \sqrt{\frac{\rho}{E_c}} = \frac{\omega}{c_L}
\]

\[
c_L = \frac{E_c}{\rho}
\]
\( \beta \) is called as wavenumber (Cremer et al., 1988), which is a measure of the phase change per unit distance of a propagating wave, and \( c_L \) is the constant axial wave velocity.

The homogeneous solution for partial differential equation is,
\[
\frac{\partial^2 u}{\partial x^2} = \beta^2 u = 0
\]

The element stiffness matrix is computed using the stiffness method, assuming linear spring behavior \( \{ f \} = [k_{col}] \{ u \} \), by first applying a unit axial displacement to the left end of the rod, and then to the right. By fixing the right end of the rod and applying a unit displacement to the left end, the coefficients \( C_1 \) and \( C_2 \) can be solved,
\[
C_1 = -\frac{\cos(\beta L)}{\sin(\beta L)} \quad \text{and} \quad C_2 = 1
\]

Therefore, the steady state displacement field for a rod fixed at one end and free at the other, similar to a founded structural column, is,
\[
u(x) = -\frac{\cos(\beta L)}{\sin(\beta L)} \sin(\beta x) + \cos(\beta x)
\]
and the corresponding force field is,
\[
f(x) = -E_c A \beta \left[ \frac{\cos(\beta L)}{\sin(\beta L)} \cos(\beta x) + \sin(\beta x) \right]
\]

By fixing the left end of the rod and applying a unit displacement to the right end, the coefficients are calculated (Brett 2007),
\[
C_1 = \frac{1}{\sin(\beta L)} \quad \text{and} \quad C_2 = 0
\]

The corresponding steady state displacement field and the force field are:
\[
u(x) = \frac{\sin(\beta x)}{\sin(\beta L)}
\]
\[
f(x) = E_c A \beta \frac{\cos(\beta x)}{\sin(\beta L)}
\]

The reactions at each end, related to each displacement, are the elements of the dynamic stiffness matrix of a single column acting as a bar element, and can be written as,
\[
[k_{col}] = \frac{E_c A \beta}{\sin(\beta L)} \begin{bmatrix} \cos(\beta L) & -1 \\ -1 & \cos(\beta L) \end{bmatrix}
\]

**Modeling of Floor Slab.** Floors attached to the columns can be modeled as energy dissipating floors slabs. The bending theory used to describe the response of the floor is thin plate theory. Thin infinite plates are totally resistive in the sense that they carry energy away from driving point (the column). This resistance is called point force impedance and is given as:
\[
Z = 8 \sqrt{D \rho h}
\]
where $D$ is the bending stiffness of a plate of thickness $h$

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)}$$ (16)

In general terms, impedance is the ratio of the applied force $f$ to the response velocity $v$,

$$Z = \frac{f}{v}$$ (17)

Thus using Newton’s second law and eq. (17),

$$f = m_{\text{eff}} a = Z v$$ (18)

The dynamic effective mass of the plate ($m_{\text{eff}}$) which is dependent on the driving frequency, is given by,

$$m_{\text{eff}} = \frac{Z}{j\omega} = \frac{j}{\omega}8\sqrt{D}\sqrt{\rho h}$$ (19)

**Assembly of System of Equations.** The dynamic stiffness matrix of the column elements, $[k_{\text{col}}]$ and the dynamic floor participation, $[m_{\text{eff}}]$ are assembled into global system matrices $[K_{\text{col}}]$ and $[M_{\text{slab}}]$, to be used in the steady state response, $\{U\}$, of a building column subjected to a harmonic loading, $\{F\}$.

$$[K_{\text{col}}]\{U\} - \omega^2[M_{\text{slab}}]\{U\} = \{F\}$$ (20)

Note that the stiffness and mass matrices are both frequency dependent in that the elements change based on the driving frequency.

The frequency dependent steady state response for all the floors becomes,

$$\{U\} = [K_{\text{col}} - j \omega^2 M_{\text{slab}}]^{-1} \{F\}$$ (21)

and the velocity is given as,

$$\{V\} = i\omega \{U\}$$ (22)

The vibration velocity response level at all floors is given in VdB scale as,

$$\text{VdB} = 20 \log_{10} \frac{v}{v_{\text{ref}}}$$ (23)

where,

$$v_{\text{ref}} = 1 \times 10^{-8} \text{ m/s}$$ (24)

Note that only the center column is considered in the mathematical model in (20). Therefore the sizes of the above system matrices are 5 x 5 (one for modeling of the column excitation at the base and four for the point impedances of the four floors).

**EXPERIMENTATION OF SCALE MODEL BUILDING**

Zhao (2009) designed and constructed a scale model building including base and 4 floors (see Figure 2). The scale model building (6’ x 4’ x 72”) has a floor height of 17” at the base and 15” for all other floors. The model consists of 9 columns in total and 4 floor slabs. Aluminum alloy of 6105-T5 strips were used for the column while the floor slabs were constructed from medium density fiberboard (MDF).
The material properties for columns and floors are summarized in Table 1.

**Table 1. Materials used and its properties**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Column</th>
<th>Floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Aluminum (6105-T5) Alloy</td>
<td>Medium Density Fiberboard</td>
</tr>
<tr>
<td>Young’s Modulus (E)</td>
<td>70.3 GPa</td>
<td>3.15 GPa</td>
</tr>
<tr>
<td>Density ((\rho))</td>
<td>2691 kg/m(^3)</td>
<td>700 kg/m(^3)</td>
</tr>
<tr>
<td>Poisson’s Ratio ((\nu))</td>
<td>0.33</td>
<td>0.18</td>
</tr>
<tr>
<td>Loss Factor ((\eta))</td>
<td>0.002</td>
<td>0.02</td>
</tr>
</tbody>
</table>

A Bruel & Kjaer Permanent Magnetic Vibration exciter type 4808 with a force rating of 112 N was used to excite the model at the base on one column. A Bruel & Kjaer force gauge type 8230 was used at the base of the column to measure the excitation force. PCB accelerometers type 352C65 were used to measure the vibrations on the floors at the center column.

The scale model building in Figure 2 consisted of base and 4 floors with accelerometers attached at each level. A Brüel & Kjær Permanent Magnetic Vibration Exciter Type 4808 was connected to the base of the center column. In order to measure the force applied to the structure, a load cell was mounted at the base of the center column. The shaker was supported by four neoprene base isolators to minimize flanking excitation to other columns. The eight (side and corner) columns that were not excited were buried in sand-filled buckets.

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The scale model building was excited axially at the base of the center column for the purpose of testing and modeling the vertical propagation of structure-borne vibrations. A white noise signal was generated as the excitation source with a 50 kHz sampling rate and a sampling block of 50,000 data points. The length of each set of data was 1 second and the frequency resolution was 1 Hz. A Hanning window was applied to each set of one-second measured data in the time domain. The measured acceleration was transformed to the frequency domain using FFT function in Matlab.

Figure 3 shows the measured excitation force after taking moving average over 201 points. The horizontal axis shows the range of frequency from 10 to 5000 Hz while the vertical axis shows the measured input force in N. A moving 201 point average was used to smooth the results.

![Force Spectrum](image)

Acceleration measurements were made in the vertical direction on the center column at each floor, as well as at the column base. The measured data obtained from the load cell gives the measurement of the applied force at the base through shaker at the corresponding frequency. These readings were used to calculate and predict the velocity response at each level using the analytical model. The measured acceleration data were used to calculate the velocity at each floor. A moving 201 point average was used to smoothen the results. The plot comparing the predicted and measured velocity response is thus obtained. Figure 4 shows that the predicted and measured velocity response at each floor and the base of the building in the same frequency range after taking the moving average over 201 points. The vertical axis shows the velocities in VdB with a reference velocity of $10^{-8}$ m/s.

The ratio of measured input force to the velocity of the floor is defined as the impedance of that floor slab. If the velocity response is at the point of excitation, it is called the driving point impedance. If the response is measured at other locations within the building, it is called the transfer impedance. The data obtained through this experiment was further used to compare the predicted impedance to the impedance measured experimentally. Figure 5 shows the predicted and measured impedances at each floor and the base of the building. The vertical axis shows impedances in dB reference to 1 N-s/m.
Figure 4. Predicted and Measured Velocity Response for Each Floor
Figure 5. Predicted and Measured Impedance for Each Floor
**Observations.** As it can be observed from the plots (Figures 4 and 5), the predicted velocities and impedances are very good predictors of the measured values. However, there are some discrepancies found in the high frequency region. Also it can be inferred from these plots that the accuracy in the prediction of velocity and impedance is better for higher floors.

**Sources of errors.** The probable reason for the deviation at the higher frequency may be due shear softening of the floor slabs at higher frequencies. The predictions are expected to roll off at higher frequencies if a thick infinite plate model (Mindlin model) was used instead of a thin plate bending theory. Mindlin model accounts for shear deformation of the cross-section of the floor. Another source of error may be due to the bending phenomena that can be present in the column.

**Impedance based model: advantages and limitations.** The impedance based model, compared to the other approaches like finite element approach (which requires many more DOF at higher frequencies with greater cost and time to characterize an extended building structure), is simple, efficient, and provides a fast way to estimate the effects of vibrations induced by passing trains or other environmental sources.

In its present form however, the impedance based model cannot be used to predict the vibration induced in a building containing a staircase or opening in the floor slab or other kind of discontinuities and irregularities.

**CONCLUSIONS AND FUTURE WORK**

The impedance based model developed was verified with experimental results from a scale model building for longitudinal excitation of a building column. From the results and the observations, it can be concluded that:

1. The impedance-based model successfully predicted the velocity response of the scale model building at column locations.
2. These predictions are more accurate in the lower frequency region as compared to the high frequency region.
3. This method can be applied accurately to special cases where the train induced vibrations are mainly vertical in transmission to building columns.
4. Impedance-based modeling was used for prediction of train-induced vibrations in buildings.
5. Wave propagation models are highly efficient in terms of computation time required.

In the future, use Mindlin thick plate theory to improve modeling of the floor behavior. This model is expected to improve the accuracy of the predictions in the high frequency region. Shear deformations need to be included in the response in addition to bending deformations. Furthermore, full scale testing of a building will be performed for validations of the presented wave propagation method.
Future work also includes plans to test a full scale building during various stages of construction and fit-out. It is well known that non-structural elements can affect vibration measurements significantly. In fact, the inability to correlate basic models with real buildings at a level of acceptable accuracy, led to this study. This completed model study, will have a significant impact on future work in real buildings, helping to direct methods of measurement, and helping to discern the different effects of structural and non-structural elements.

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