Predicting Train-Induced Vibrations in Multi-Story Buildings

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ABSTRACT

Urban societies face the challenge of working and living in environments filled with noise and vibration caused by construction, manufacturing, and transportation systems. Due to soaring prices of real estate in modern cities, air-rights developments are becoming more widespread. As residential towers and hotels are built over or adjacent to train stations, subways, and highways, traffic noise and vibration play an ever increasing role in causing discomfort to the occupants. Research laboratories conducting highly sensitive measurements and manufacturing plants for nanotechnology are also prone to disruption by feelable and audible vibrations. In this research, a wave propagation model is developed to predict train-induced vibrations in buildings. The analytical predictions are compared to experimental data obtained using an electrodynamic shaker in a full-scale building. The floor responses to controlled shaking are used to calibrate the wave propagation model by estimating model parameters using optimization techniques. Train-induced vibrations, introduced at the building foundation, are also measured and compared to analytical predictions.

INTRODUCTION

Societies with ever-growing urban populations face the daily challenge of working and living in environments filled with noise and vibration caused by construction, manufacturing, and transportation systems. Congested metropolitan areas with high land values are more frequently building over air-rights of rail transportation systems or are located in their proximity. Vibrations from these sources can propagate through the ground and into the occupied space of buildings, disturbing occupants and sensitive manufacturing or laboratory equipment. Significant construction costs could be avoided if predictive models were available to discern at the outset whether or not site vibrations might be problematic in proposed buildings.

The TD Banknorth Garden (Figure 1 - Courtesy of TD Banknorth Graden) is a nine story composite steel frame events arena with cast-in-place concrete floors (normal weight and light weight) on metal deck. The floor slabs vary in thickness and density from floor to floor and the steel framing is of a non-uniform layout (attributed to the presence of both rectangular and radial gridlines). The building is a hub for metropolitan sports and cultural events and hosts the
Massachusetts Bay Transportation Authority (MBTA) North Station commuter rail and subway lines. In addition to the entertainment activities, the building also contains office and retail space.

Current guidelines for predicting train-induced vibrations in buildings, published by the FTA (Harris Miller, 2006), rely on a heuristic predictive model. The FTA’s recommendations for estimating floor-to-floor vibration attenuation are -2dB/floor (1 to 5 floors above grade) and -1 dB/floor (5 to 10 floors above grade). The FTA manual also recognizes that some floors may exhibit resonant behavior, amplifying vibrations by up to 6 dB. There have been several attempts to develop predictive finite element models, but the models tend to be cumbersome and too computationally intensive for practical use (Lee et al., 2000).

The prediction of train-induced vibrations in buildings is a potentially arduous undertaking considering the complexity of civil structures, limitations of data acquisition systems, the inadequacy of modeling procedures, and constraints with parameter estimation. This research identifies fundamental elements from which a predictive model can be built then calibrated with field measurements.

**AMBIENT AND TRAIN INDUCED VIBRATIONS**

The MBTA Green Line Subway runs beside a slurry wall that supports Column V.1-1.5. From its base, this column runs up to the third floor where it makes a 0.76m transfer to Column V.1-1.53. Column V.1-1.53 runs up from the third floor before terminating at the seventh floor. This column was a primary candidate for study since: (1) it is relatively accessible on most floors for instrumentation, (2) its proximity to the Green Line track make it an excellent receiver of train-induced vibrations, and (3) it is easy to excite the top of the column with a portable shaker, since it terminates on the seventh floor.

A mobile vibration measurement system, developed at Tufts University (Dodge, 2006; Brett, 2007) was used to measure ambient and train-induced vibrations on the bottom seven floors of the TD Banknorth Garden - the reader should note that because of an instrument malfunction, data are not available for the fourth floor. Measuring ambient vibrations established a vibration baseline on each floor, and train-induced vibration measurements were used to confirm the accuracy of the predictive model developed in this research.

One-third-octave analysis is used to analyze the vibration signals. In such an analysis, the time domain vibration signal is passed through a series of band-pass filters whose upper and lower frequency bands are defined by the American National Standards Institute (ANSI, 2004). The spectral vibration magnitudes within the pass region are summed and assigned to a band that is identified by the filter's center frequency (20, 25, 31.5, 40, 50, 63, 80, 100, 125, 160, 200, 250 or 315 Hz, etc.). This process condenses the data into standard, widely used, frequency bands. The FTA Noise and Vibration Manual (Harris Miller, 2006) describes how spectral velocity measurements can then be expressed in terms of vibration velocity level (VdB), as in Equation (1), where \( v_{ref} \) is a reference velocity of 1μm/sec.

\[
VdB = 20 \log \left( \frac{v}{v_{ref}} \right)
\]  

(1)

Vibrations need not be feelable to be perceived by building occupants. The vibrating walls, floors and ceilings also radiate sound like giant loudspeakers. This radiated noise is often perceived as a low-frequency “rumble” when a train passes by. Since the building vibration is directly related to radiated noise, it is common to apply an A-weighting correction (ANSI, 2004) to the vibration spectra in order to account for the frequency-dependent sensitivity of human
hearing. The FTA manual provides a method to calculate the expected radiated noise levels from the floor vibration velocity. The lower frequency limit of human audible perception is around 20 Hz, and therefore also is taken as the lower bound of measurements presented here.

Ambient and Train-Induced Vibration Measurements (One-Third-Octave Spectra) - Table 1 shows the total vibration and equivalent audible noise magnitudes obtained for the lowest seven floors (calculated as the energy sum of the one-third-octave levels between 20 Hz and 315 Hz, with respect to a reference velocity of 1 μin/sec). For reference, the recommended FTA limits for office space subjected to train-induced vibrations are 75 VdB and 40 dBA for vibration and noise magnitudes, respectively. While the measured feelable vibrations were all below the FTA recommended limits for human comfort, the noise on the first floor of the TD Banknorth Garden exceeded the corresponding recommended limit. However, the first floor is a public mezzanine and commuter rail platform, and therefore is exempt from such criteria. This building exhibited losses of 2.4 VdB and 3.7 VdBA between the first and second floor, and 7.2 VdB and 12.8 VdBA between the second and third floors, for vibration velocity level and equivalent audible noise level, respectively. These losses differed significantly from the 2 dB loss per floor prescribed by the FTA Manual. The change per floor in measured vibration velocity level cannot be compared to the FTA recommendations above the third floor because the ambient vibrations on the higher floors mask the train-induced vibrations.

### Table 1 - Total Noise and Vibration Level Measurements During Train Passage

<table>
<thead>
<tr>
<th>Floor</th>
<th>Vibration Velocity Level (VdB)</th>
<th>Change Per Floor (VdB)</th>
<th>A-Weighted Noise Level (VdBA)</th>
<th>Change Per Floor (VdB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>59.8</td>
<td>N/A</td>
<td>42.7</td>
<td>N/A</td>
</tr>
<tr>
<td>Second</td>
<td>57.4</td>
<td>-2.4</td>
<td>39.0</td>
<td>-3.7</td>
</tr>
<tr>
<td>Third</td>
<td>50.2</td>
<td>-7.2</td>
<td>26.2</td>
<td>-12.8</td>
</tr>
<tr>
<td>Fifth</td>
<td>48.3</td>
<td>-1.9</td>
<td>28.4</td>
<td>2.2</td>
</tr>
<tr>
<td>Sixth</td>
<td>49.7</td>
<td>1.4</td>
<td>24.0</td>
<td>-4.4</td>
</tr>
<tr>
<td>Seventh</td>
<td>48.3</td>
<td>-1.4</td>
<td>22.2</td>
<td>-1.8</td>
</tr>
<tr>
<td>FTA Limits</td>
<td>75</td>
<td></td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

Figures 2 through 8 show the measured ambient and train-induced vibrations. These figures also contain analytical predictions that will be discussed later. The one-third-octave-band vibration velocity levels in VdB are A-weighted to permit relation of the corresponding sound to the perception of the human ear. An attenuation trend is visible, as the train-induced vibrations are much greater than ambient levels on the first floor and then gradually approach the ambient vibration levels by the fifth floor. Since the fourth floor data is unavailable, it is not clear what happened to the train-induced vibrations between the third and fifth floors. However, the trend of vibration attenuation on the remaining floors appears strong, so this data set can still be used to validate train vibration predictions.
SHAKER INDUCED VIBRATIONS

While ambient and train-induced vibrations represent actual conditions, they do not originate from a source that is controlled or easily quantified. For this reason, an APS Electro-seis 400 electrodynamic shaker was used to provide harmonic excitations for a range of discrete frequencies on the highest instrumented floor, while the vertical transducer array captured the steady state response of the floors below.

The shaker was placed directly atop Column V.1-1.53, at the seventh floor (located by triangulation ties to building features). An array of vertically oriented accelerometers was installed on each floor below the shaker. Four stepped sine-wave tests were conducted by dwelling at discrete frequencies between 10 Hz and 300 Hz. These tests were used to expedite the execution of a suite of harmonic tests, driving the structural system to steady state and maximizing the signal-to-noise ratio. A number of samples of each stepped sine sweep test were gathered for frequency-domain averaging. Averaging minimizes signal contamination caused by transients. The coherence function was used to evaluate the quality of the measurements on all floors, providing an indication of when a sufficient number of samples had been gathered (National Instruments, 2005).

The ratio of the response velocity magnitude to the excitation velocity magnitude (velocity ratio) was computed for each floor using Equation (1), with $v_{ref}$ taken as the spectral velocity magnitude of the excitation floor at the drive location ($7^{th}$ floor). The slab thickness was used as the initial rough value of the floor thickness. The raw processed velocity ratio measurements appear disorderly to some extent, but major trends (peaks and valleys in the frequency response) become apparent when a four-point running average filter is used to smooth the plotted measured data (markers without lines) shown in Figure 9. Note: This figure also contains analytically obtained velocity ratios (markers connected with lines), discussed later.

![Figure 9 - VELOCITY RATIOS: ANALYTICAL & MEASURED](image)

Figure 10 – SEVEN STORY MODEL

ANALYTICAL MODELING

A mathematical model for predicting the propagation of train-induced vibrations in buildings was developed, Figure 10. In this model, structural columns are modeled as wave-propagating rods and floors are modeled as infinite plates that conduct energy away. (Brett, 2007).
**Column Modeling (Wave Propagation Theory)** - The element dynamic stiffness matrix is computed assuming linear dynamic stiffness behavior, \( \{f\} = [k_{col}]\{u\} \).

\[
[k_{col}] = \frac{EA\beta}{\sin(\beta L)} \begin{bmatrix} \cos(\beta L) & -1 \\ -1 & \cos(\beta L) \end{bmatrix}
\]  

(2)

Where \( \beta \) is the wavenumber (Cremer et al., 1988):

\[
\beta = \omega \sqrt{\frac{\rho}{E}}
\]  

(3)

In this formulation the element stiffness matrix is frequency-dependent, where \( \omega \) = circular frequency, \( \rho \) = mass density, and \( E \) = modulus of elasticity. The complex modulus is used to include material damping as follows (Fahy, 1985)

\[
E_c = E(1 + i\eta)
\]  

(4)

where the material loss factor, \( \eta \), is proportional to the energy dissipated by the column during each cycle of vibration; for small damping, the loss factor is approximately equal to \( 2\varsigma \), twice the viscous damping ratio.

**Floor Modeling (Infinite Plate Theory)** - Floors are taken into account via the driving point impedance of corresponding infinite plates. This introduces the notion of the flow of vibration energy from the columns outward along the floor. The point force impedance of an infinite plate obeys

\[
Z = 8\sqrt{D} \sqrt{\rho h}
\]  

(5)

Where \( \rho h \) represents plate mass per unit area, and \( D \) is the plate bending modulus (\( h \) = plate thickness and \( \nu \) = Poisson’s ratio, and \( E \) = modulus of elasticity)

\[
D = \frac{Eh^3}{12(1-\nu^2)}
\]  

(6)

Solving for \( m_{eff} \) using the complex form of the frequency domain relationship between acceleration and velocity (\( v = a/\omega \)) yields a lumped, frequency-dependent, imaginary “mass”

\[
m_{eff} = \frac{Z}{i\omega} = -\frac{i}{\omega} 8\sqrt{D} \sqrt{\rho h}
\]  

(7)

**System Assembly and Solution** - The mass and stiffness of column elements, \([k_{col}]\), and the impedances of floors, \( m_{eff} \), are assembled into global system matrices \([K_{col}]\) and \([M_{eff}]\), similar to finite element methods, and used in the equation for steady state response of an axial array of elements subject to harmonic loading,

\[
[K_{col}]\{U\} - \omega^2[M_{eff}]\{U\} = \{F\}
\]  

(8)

where there is one vertical degree of freedom at each node. Recall that the stiffness matrix elements are both complex and frequency dependent.
WAVE PROPAGATION MODEL OF A BUILDING COLUMN

In this formulation, only a single column is modeled, vibration contributions from adjacent columns are neglected. In order to determine the elements of the global mass and stiffness matrices member sizes and structural dimensions are obtained from structural and architectural drawings. For a seven story column axial wave propagation model; \([M]\) and \([K]\) are 7x7 matrices and \(\{F\}\) is a 7x1 vector. The foundation stiffness is estimated and added to the first element of the global stiffness matrix in order to provide a boundary condition. For shaker excitation on the 7th floor the force vector is all zeros except for the last element, which is one. This represents an axial unit force applied to the column at the top floor at each frequency of interest. Equation (8) is used to solve for the vertical displacement response of all floors to a harmonic unit load acting on the upper floor of the seven-story model. In this manner, analytical displacements are obtained for the same frequencies used in the shaker dwells. The RMS displacement magnitudes are converted to velocity and expressed as a logarithmic ratio relative to the velocity of the floor where the load is applied, using Equation (1) with \(v_{ref}\) taken as the velocity of the floor directly driven by the shaker).

Figure 9 shows analytical velocity ratios based on contract drawing geometry and member sizes (markers connected with lines) superimposed on the stepped sine sweep measured data (markers without lines). The sine sweep data points shown are smoothed with a 4-point running average filter. The analytical velocity ratios exhibit two distinct peaks: one just below 100 Hz and another around 185 Hz. These peaks represent the vertical inter-story resonant modes of vibration (higher than the fundamental mode of the cantilever column). The lower response for floors farther away from the vibration source is apparent as the plots for each floor are ordered in descending fashion at lower frequencies. The model behavior is more complex at higher frequencies as higher modes come into play. The superimposed figures indicate that the model, in its original state, is not adequate for predicting the floor-to-floor attenuation of vibrations.

PARAMETER ESTIMATION USING SHAKER-INDUCED VIBRATION MEASUREMENTS

A sensitivity analysis was performed to identify the key parameters that influenced the analytical velocity ratios (Brett, 2007). In this analysis, key parameters, such as column cross-sectional areas, column loss factors, and floor thicknesses were changed systematically for each story and response sensitivities are observed.

Coherence measures were used to weight the error function, thereby allowing the data with greater effect to have greater influence in directing the parameter estimation procedure.

Figure 11 - VELOCITY RATIOS: OPTIMIZED & MEASURED
The Gauss-Newton method of optimization was then used to estimate the parameters of the analytical model. Convergence was considered achieved when $J(p)$ was at a minimum and changes less than 0.05% with respect to the previous iteration (Sanayei and Onipede, 1991).

**Optimized Parameters for the TD Banknorth Garden** - The parameters estimated by optimization procedure, shown in Table 2 enable more accurate numerical assessment of the building’s physical behavior. This does not guarantee, however, that these parameters represent actual physical characteristics of the building. Figure 11 shows the analytical velocity ratios relative to floor 7, computed with the optimized parameter values (markers connected with lines) and superposed on the measured velocity ratios (markers without lines). Figure 11 indicates a reasonably good match between the optimized velocity ratios and the measured velocity ratios.

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Initial Parameter</th>
<th>Estimated Parameter Ratio</th>
<th>Final Parameter Estimate</th>
<th>Calculated Effective Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{slab}$ 1st floor</td>
<td>2.00 m</td>
<td>0.22</td>
<td>0.44 m</td>
<td>N/A</td>
</tr>
<tr>
<td>$h_{slab}$ 2nd floor</td>
<td>0.12 m</td>
<td>4.71</td>
<td>0.57 m</td>
<td>0.94 m</td>
</tr>
<tr>
<td>$h_{slab}$ 3rd floor</td>
<td>0.22 m</td>
<td>6.26</td>
<td>1.38 m</td>
<td>0.72 m</td>
</tr>
<tr>
<td>$h_{slab}$ 4th floor</td>
<td>0.17 m</td>
<td>1.71</td>
<td>0.29 m</td>
<td>0.54 m</td>
</tr>
<tr>
<td>$h_{slab}$ 5th floor</td>
<td>0.12 m</td>
<td>5.08</td>
<td>0.61 m</td>
<td>0.61 m</td>
</tr>
<tr>
<td>$h_{slab}$ 6th floor</td>
<td>0.12 m</td>
<td>1.82</td>
<td>0.22 m</td>
<td>0.35 m</td>
</tr>
<tr>
<td>$A_{col}$ 1-2 Column</td>
<td>0.442 m²</td>
<td>0.25</td>
<td>0.111 m²</td>
<td>N/A</td>
</tr>
<tr>
<td>$A_{col}$ 2-3 Column</td>
<td>0.442 m²</td>
<td>0.18</td>
<td>0.08 m²</td>
<td>N/A</td>
</tr>
<tr>
<td>$A_{col}$ 3-4 Column</td>
<td>0.171 m²</td>
<td>0.87</td>
<td>0.149 m²</td>
<td>N/A</td>
</tr>
<tr>
<td>$A_{col}$ 4-5 Column</td>
<td>0.171 m²</td>
<td>0.27</td>
<td>0.046 m²</td>
<td>N/A</td>
</tr>
<tr>
<td>$A_{col}$ 5-6 Column</td>
<td>0.091 m²</td>
<td>0.86</td>
<td>0.078 m²</td>
<td>N/A</td>
</tr>
<tr>
<td>$A_{col}$ 6-7 Column</td>
<td>0.091 m²</td>
<td>0.24</td>
<td>0.022 m²</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**TABLE 2 – FLOOR THICKNESS AND COLUMN AREA PARAMETER VALUES**

The estimated floor thicknesses reported in Table 2 are substantially greater than the actual building slab thicknesses; with the first floor being the exception (its thickness was initially overestimated). The fact that the floors were able to dissipate more vibration energy than initially assumed led to the hypothesis that stiff beams and transfer girders framing into Column V.1-1.5 had not been appropriately considered in the initial slab model. To quantify the impedance effect of these framing elements, an effective concrete slab thickness was estimated by considering the composite effect of the steel beams/girders supporting the slab and back-calculation an equivalent effective thickness of a concrete slab of uniform thickness, shown in the last column of Table 2. In some cases the estimate of effective thickness is close to the final parameter value, but in others it is not. The steel framing beneath the slab is quite irregular and complex, making the effective slab thickness difficult to calculate, but the preliminary estimates of uniform slab thickness indicate that the optimized thicknesses may, in fact, represent several paths of vibration energy loss from Column V.1-1.5 (Figure 10), such as losses into the floor slab, beams, girders, adjacent columns, and nonstructural elements. For structures where measurements are not possible, a more detailed analysis of the floor system may help to estimate an effective uniform slab thickness in advance, since the velocity ratios produced by using just the concrete slab thickness and equivalent effective uniform floor thickness are not as accurate as those produced by the optimization. The column area ratios are all below 1.0, indicating that not all of the column area may be used for the transmission of vibration energy. The necessity to use lesser
column areas is not quite clear, but it may be a result of the relative ability of the floor to remove great amounts of vibration energy from the column. The models with updated parameters as shown superimposed on measurements, in Figure 11 are a significant improvement over the model with initial parameters shown in Figure 9. The analytical velocity ratios for each floor capture the major trends in the measurements, providing a much better fit.

TRAIN-INDUCED VIBRATION PREDICTION USING UPDATED PARAMETERS

The updated parameter model, now with a unit harmonic load applied to the bottom floor rather than the top, was used to establish the vibration relationship between floors due to a vibration source at the foundation. Since the ambient and train-induced vibration measurements were quantified at third-octave center frequencies, the analytical velocity ratios were computed at the same frequencies. Because the model was calibrated with shaker-induced vibration measurements at frequencies up to 300 Hz, use of the model for the purpose of predicting train-induced vibrations was therefore limited to the one-third-octave frequency range of 20 - 315 Hz.

Figure 12 shows the updated analytical velocity ratios at one-third-octave center frequencies, relative to the bottom floor (v1). The relative vibration velocity levels in this figure are used to predict train-induced vibrations on upper floors by summing the measurements made on the lowest floor with the corresponding analytical values for each floor in dB. The train-induced vibration predictions for upper floors, based on this method, are shown in Figures 3 through 8.

The second floor predictions, Figure 3, match measurements quite well below 60 Hz. Between 60 Hz and 110 Hz the predictions stray from the measurements, but appear to be an average representation of the train-induced vibration data. The model does not capture the attenuation of high frequencies between the first and second floor above 200 Hz.

The third floor predictions, Figure 4, have nearly the same shape as measurements made on that floor but are consistently lower, except at 100 Hz. Though all measurements made on the fourth floor have been omitted, the final model shown in Figure 12 is representative of the entire system, including the fourth floor. Train-induced vibration predictions can therefore still be made on the fourth floor, but not substantiated by measurements. Figures 5 to 8 indicate that most of the train-induced vibrations on these floors do not exceed the ambient vibrations.

CONCLUSIONS

Train-induced vibrations were measured on the bottom seven floors in the TD Banknorth Garden. The total vibration levels and floor to floor attenuation characteristics were compared to FTA methods, whereby it was discovered that the FTA estimate of a 2 dB loss per floor was inadequate for predicting vibration attenuation in the TD Banknorth Garden.
A structural column spanning several floors was modeled as a series of wave-propagating and frequency-dependent rods. Floors were modeled as infinite plates attached to the columns at each floor level. Comparison of the initial analytical floor velocity ratios to the measured velocity ratios indicated that the idealized model did not produce velocity ratios that were similar to the measured data. The Gauss-Newton method of optimization was then used to estimate the key parameters of the analytical model and the result was successfully validated by superimposing the updated analytical velocity ratios and the measured velocity ratios. The updated parameters was generally corroborated with the building drawings.

The seven-story wave propagation model, with updated parameters, successfully predicted train-induced vibrations on the first two floors above the foundation. Train-induced vibrations were significantly masked by ambient levels after the bottom three floors. This was confirmed by field measurements and by the researchers' observations during testing.

Floor mass per unit area, \( \rho h \), contributes some to vibration confinement and dissipation, but floor bending stiffness, \( D \), has the most influence on energy dissipation and vibration confinement. This is an important realization for practicing structural engineers since a transfer floor, often required for other purposes, can be a cost effective solution for vibration mitigation of higher floors.

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