Impact of Finite Element Modeling Error on Model Updating

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ABSTRACT

Unlike in conventional structural analysis, modeling error has a significant impact when performing parameter estimation for model updating, health monitoring, damage assessment, and structural control. Modeling error is introduced when there are uncertainties in the assumptions of the analytical model. Two types of modeling error are studied: local modeling error representing an undetected damage and global modeling error representing uncertainty in material properties. A 2D-bridge bent is modeled and the impact of local and global modeling error on structural stiffness parameters is studied through simulations. Performance of the ‘flexibility-based modal error function’ and the ‘stiffness-based modal error function’ are compared. The performance of the latter error function was in general superior in the presence of modeling error. In addition, the quality of the structural parameter estimates are dependent on the topology of the structure, different combinations of frequency and mode shape measurements, and type and magnitude of modeling error.

1. INTRODUCTION

Structural parameter estimation is a powerful tool enabling the prediction of structural stiffness and mass parameters for model updating. Finite element models (FEM) are often used to capture the analytical response of structures. A major challenge in parameter estimation is to reduce error in mathematical modeling. Modeling error is introduced when assumptions of an analytical model are not entirely correct. Major areas of modeling error are the existence of nonstructural members; uncertainty in material, stiffness, and mass properties of known parameters; incomplete information about boundary conditions; nonlinear structural response; modeling of energy losses (damping); and environmental variability for parameter estimation (Sanayei et al.1998a). In direct structural analysis, the impact of modeling error is predictable and easily understood by practicing engineers. However, modeling error can have a significant impact when performing parameter estimation for model updating, health monitoring, damage, and structural control.
Foundations, commonly modeled as fixed or pinned, are one of the major components of a structure that ordinarily are not captured accurately. To reduce the loss in accuracy of the parameter estimates due to boundary condition modeling error, a soil-substructure super-element is used to capture the lumped stiffness and mass properties of the soil-structure interface (Sanayei et al. 1998b; McClain 1996). As the size of the FEM increases, so does the number of unknown parameters that are required to develop an understanding of the general state of the structure. In order to maximize the use of the measurements and minimize computational time, unknown parameters that are of the same nature and identical are grouped to represent a single stiffness parameter (Sanayei et al. 1998b; McClain 1996).

There are three categories of parameters: known, unknown, and uncertain. The known parameters are those parameters in which the user holds a relatively high level of confidence based on the available information such as blueprints and field measurements. The unknown parameters are adjusted through the parameter estimation process. The uncertain parameters are considered known with some level of uncertainty (Gunes et al. 1998; Wadia-Fascetti et al. 1998). A simulated study of the effect of uncertain parameters on the estimated values of unknown parameters provides a tool for understanding the behavior of the model before field data is used for parameter estimation. Two types of modeling error are studied: local and global. Local modeling error exists due to complexity of interfaces such as foundations and connections, or due to an undetected local damage. Global modeling error exists due to uncertainty in material and/or section properties that impact the entire structure.

PARIS (PARameter Identification System) computer program is used to investigate the feasibility of parameter estimation (PARIS® 1997). In order to gain an understanding of the behavior of the FEM for parameter estimation in the presence of error, a modeling error study is conducted. The performance of the 'stiffness-based modal error function' (Sanayei et al. 1998b; Gornshytn 1992) and the 'flexibility-based modal error function' which is recently developed for this research is compared in presence of modeling error.

An example is given to show the impact of modeling error on model updating using simulated modal data. Olson Engineering performed a dynamic non-destructive test (NDT) on a full-scale bridge in Liberty County, Texas, for foundation stiffness identification (Aouad et al. 1998). In preparation of using this data, one 2D-bridge bent is modeled and the impact of local and global modeling error is studied through simulations (Santini 1998).

2. PARAMETER ESTIMATION FOR STRUCTURAL IDENTIFICATION

Structural parameter estimation is the solution of the inverse problem to reproduce the measured response by adjusting the stiffness parameters of the analytical model of the structural system at the element level. Although the theory behind parameter estimation is not new, increased computational capability has resulted in significant progress in algorithm development and testing. Mottershead and Friswell (1993), Ghanem and Shinozuka (1995), Shinozuka and Ghanem (1995), Doebling et al. (1996) survey the current state-of-the-art of structural identification methods. PARIS estimates the stiffness and mass parameters, \( \{ p \} \), of structures to match the analytical response with the measured NDT data. Examples of parameters are axial, bending and torsional rigidities as well as the effective lumped stiffness and mass properties of foundations. The measured NDT data used may be static or modal (Sanayei et al. 1997; 1998b). The 'stiffness-based modal error function' and the 'flexibility-based modal error function' are used to estimate the parameters of a linear-elastic structure's FEM at the element level. This can easily lend itself to damage assessment of structures.
2.1. Stiffness-Based Modal Error Function

The characteristic equation for dynamic loading (1) can be partitioned in terms of mode shape coordinates at measured and unmeasured DOF for each selected natural frequency as shown in (2) (Sanayei et al. 1998b; Gornstheyn 1992). This partitioning is necessary for large FEMs when mode shape coordinates are not measured at all DOF.

\[
[K]\{\Phi\}_j = \lambda_j[M]\{\Phi\}_j
\]

\[
\begin{bmatrix}
K_{aa} & K_{ab} \\
K_{ba} & K_{bb}
\end{bmatrix}
\begin{bmatrix}
\Phi_a \\
\Phi_b
\end{bmatrix}_j = \lambda_j
\begin{bmatrix}
M_{aa} & M_{ab} \\
M_{ba} & M_{bb}
\end{bmatrix}
\begin{bmatrix}
\Phi_a \\
\Phi_b
\end{bmatrix}_j
\]

In (1) and (2), \([K]\) is the stiffness matrix of the structure, \([M]\) is the mass matrix, \(\{\Phi\}_j\) contains the \(j^{th}\) mode shape, and \(\lambda_j\) is the \(j^{th}\) Eigenvalue. The subscripts 'a' and 'b' denote measured and unmeasured DOF, respectively. By condensing out the unmeasured mode shapes, \(\{\Phi_b\}\), the residual 'stiffness-based modal error function', \(\{e_s(p)\}_j\), is obtained, where \(\{p\}\) is the vector of the unknown parameters. This equation uses only measured DOF, which is essential for effective and practical parameter estimation.

\[
\{e_s(p)\}_j = \\
((K_{aa} - \lambda_j[M_{aa}]) - (K_{ab} - \lambda_j[M_{ab}])((K_{bb} - \lambda_j[M_{bb}])^{-1}(K_{ba} - \lambda_j[M_{ba}]))\{\Phi_a\}_j
\]

2.2 Flexibility-Based Modal Error Function

The characteristic equation (1) can be rewritten as (4) to use the flexibility matrix rather than the stiffness matrix by using the dynamic matrix \([D] = [K]^{-1}[M]\).

\[
\{\Phi\}_j = \lambda_j[D]\{\Phi\}_j
\]

Similar to the stiffness-based solution, (4) is partitioned in terms of mode shapes at measured and unmeasured DOF for each selected frequency as shown in (5).

\[
\begin{bmatrix}
\Phi_a \\
\Phi_b
\end{bmatrix}_j = \lambda_j
\begin{bmatrix}
D_{aa} & D_{ab} \\
D_{ba} & D_{bb}
\end{bmatrix}
\begin{bmatrix}
\Phi_a \\
\Phi_b
\end{bmatrix}_j
\]

Again, the unmeasured mode shapes, \(\{\Phi_b\}\), are condensed out to form the residual 'flexibility-based modal error function', \(\{e_f(p)\}_j\), as shown (6).

\[
\{e_f(p)\}_j = \lambda_j[D_{aa}] + \lambda_j^2[D_{ab}][(I - \lambda_j[D_{bb}])^{-1}[D_{ba}]\{\Phi_a\}_j - \{\Phi_a\}_j
\]

Each error function vector in (3) and (6) is horizontally appended for measured modes to form the modal error matrices \([E_s(p)]\) and \([E_f(p)]\), respectively. The resulting matrices are of size \(NMDOF\) by \(N\), where \(NMDOF\) is the number of modal measurements at selected DOF and \(N\) is the number of measured modes of vibration. The scalar objective function \(J(p)\) is defined as the Euclidean norm of the modal error matrices \([E_s(p)]\) and \([E_f(p)]\), where \(i\) represents a measurement at a selected DOF and \(j\) represents a measured mode of vibration. \(J(p)\), is minimized to estimate the stiffness parameters, \(\{p\}\), using PARIS.
At least, one independent measurement is required to estimate each unknown parameter. The change in the structural parameters to be estimated must be observable by measured modes of vibrations.

3. BRIDGE EXAMPLE

The bridge tested by Olson Engineering consisted of 18-foot span concrete decks directly supported by concrete cap beams and columns (Aouad et al. 1998; Santini 1998). A 2D-frame model of the bridge bent is shown in Figure 1. The marked nodes shown in figure 1 indicate the location of the bi-directional accelerometers on the bridge bent.

![Figure 1. Bridge bent model.](image)

**Table 1. Material and true section properties of the 2D bridge bent model.**

<table>
<thead>
<tr>
<th>Element</th>
<th>$A$ in$^2$ (m$^2$)</th>
<th>$I$ in$^4$ (m$^4$)</th>
<th>E P$si$ (MPa)</th>
<th>$\rho$ lb$*s^2$/in$/in^3$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{b1}$</td>
<td>348 (0.225)</td>
<td>37592 (0.016)</td>
<td>$5.3<em>10^6$ (3.7</em>10$^4$)</td>
<td>$2.4<em>10^{-2}$ (2.6</em>10$^4$)</td>
</tr>
<tr>
<td>$I_{b2}$</td>
<td>588 (0.379)</td>
<td>29412 (0.012)</td>
<td>$5.3<em>10^6$ (3.7</em>10$^4$)</td>
<td>$1.5<em>10^{-3}$ (1.6</em>10$^4$)</td>
</tr>
<tr>
<td>$I_c$</td>
<td>210 (0.135)</td>
<td>3490 (0.0015)</td>
<td>$5.3<em>10^6$ (3.7</em>10$^4$)</td>
<td>$2.17<em>10^{-4}$ (2.9</em>10$^3$)</td>
</tr>
</tbody>
</table>
A 3D FEM of the bridge was created in ANSYS® (1992) to verify the frequencies and mode shapes of the simplified 2D model. Moments of inertia for the groups of the structural components labeled as I_{b1}, I_{b2} and I_{c} are assumed the same for all members in each group. The density of the I_{b1} and I_{b2} were adjusted to reflect the tributary mass of the slab. In order to account for the in-plane stiffness of the slab, horizontal springs were added to the bent. The stiffness of this spring, K_{s} was determined to be 6.1*10^6 lb/in (1.07*10^9 N/m). The modulus of elasticity of the structure was determined through ultra-sonic pulse velocity tests, which reflects the tangent modulus at low stress levels hence E is greater than the values typically used in design. Using the blueprints and field data provided by Olson Engineering, section and material properties for the cap beam and piers of the bridge bent are calculated and shown in Table 1.

The foundation system consists of single concrete piles of 16 in (0.41 m) diameter and 216 in (5.49 m) long. The shear modulus of the soil of 2.2*10^3 psi (15.17 MPa) is estimated from in-situ shear wave velocity tests. Each pile was modeled with lumped springs to reflect the soil-structure interaction. A Poisson's ratio of 0.4 was used to calculate the horizontal stiffness, K_{hh} = 3.4*10^5 lb/in (5.95*10^7 N/m), vertical stiffness, K_{vv} = 6*10^5 lb/in (1.1*10^8 N/m) and the rotational stiffness, K_{00} = 6.4*10^8 lb*in (7.23*10^7 N*m) using Poulos and Davis (1980). Figure 2 shows the first four modes of vibration using ANSYS®.

![Figure 2](image)

(a) Mode 1, f = 14.067 Hz  
(b) Mode 2, f = 14.267 Hz  
(c) Mode 3, f = 21.649 Hz  
(d) Mode 4, f = 31.496 Hz

**Figure 2.** Mode shapes for the 2D FEM of the bridge bent.

There are three types of parameters in the estimation process (Gunes et al. 1998):
- **known** parameters with a high degree of confidence,
- **unknown** parameters that need to be estimated, and
- **uncertain** parameters that are assumed known in the absence of reliable data.

Modeling error complicates the solution of the inverse problem and can not be avoided in the practical application of parameter estimation. Two possible types of modeling error are studied on the bridge bent shown in Figure 1:
- **local** modeling error representing undetected damage, and
- **global** modeling error representing uncertainty in material and/or section properties.
For this example, parameter estimation is done using both modal error functions and two cases of measurements. Table 2 illustrates the measurement scenarios.

<table>
<thead>
<tr>
<th>Measurement Scenarios</th>
<th>Modes</th>
<th>Measured DOF at Selected Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (a): 2 Modes &amp; 26 DOF</td>
<td>1,2</td>
<td>All Translations of Nodes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,7,13,19,25,29,32,37,40,45,48,53,56</td>
</tr>
<tr>
<td>Case (b): 2 Modes &amp; 26 DOF</td>
<td>3,4</td>
<td>All Translations of Nodes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,7,13,19,25,29,32,37,40,45,48,53,56</td>
</tr>
</tbody>
</table>

For the parameter estimation used in this paper, element cross sectional area (A), length (L), modulus of elasticity (E), and density (p) of the cap beam and columns are assumed to be known with a high degree of certainty for the entire bridge bent. PARIS is used to implement parameter estimation that updates the structural element section properties: moments of inertia for the cap beam and columns, and the lumped spring constants used for the foundation element. The impact of the location and magnitude of each type of modeling error is quantified to determine the level of influence on the estimated parameters.

3.1 Influence of Global Modeling Error on Estimated Parameters

Global modeling error is simulated by introducing different levels of modeling error to all uncertain parameters of the bridge model. Figure 3 shows the percent error in parameter estimates for three grouped moments of inertia \( I_{b1} \), \( I_{b2} \), and \( I_c \) where modeling error is introduced to all four foundation stiffnesses, \( K_{bb} \), \( K_{vv} \), and \( K_{bb} \). Figure 4 shows the percent error in parameter estimates for \( K_{bb} \), \( K_{vv} \), and \( K_{bb} \) in four grouped foundations where modeling error is introduced to all moments of inertia of the cap beam and the four columns. In both figures, for Case (a) modes of vibration 1 and 2 are measured while for Case (b) modes 3 and 4 are measured. For ease of comparison, each parameter is given the same amount of modeling error (i.e., from -50% to +50%). MDOF denotes the degrees of freedom at which measurements are made. In all of the following figures, legends ‘o’ and ‘x’ represent the stiffness-based and flexibility-based modal error functions, respectively.

3.2 Influence of Local Damage on Estimated Parameters

Local damage is simulated on the bridge bent by adding modeling error to \( K_{bb} \), \( K_{vv} \), and \( K_{bb} \) of the right side foundation. The impact on the estimated moments of inertia \( I_{b1} \), \( I_{b2} \), and \( I_c \) and the left three foundation stiffnesses \( K_{bb} \), \( K_{vv} \), and \( K_{bb} \) are observed separately. Figure 5 shows the percent error in parameter estimates for three grouped moments of inertia \( I_{b1} \), \( I_{b2} \), and \( I_c \) while the left three foundation stiffnesses were considered known. Figure 6 shows the percent error in parameter estimates for \( K_{bb} \), \( K_{vv} \), and \( K_{bb} \) in left three grouped foundations, while all section properties of the cap beam and the columns are considered known. The right foundation stiffnesses \( K_{bb} \), \( K_{vv} \), and \( K_{bb} \) are given the same level of modeling error from -10% to -100% where -100% indicates complete failure of the right foundation.
$o =$ stiffness-based modal error function; $x =$ flexibility-based modal error function

Case (a) Measured Modes = 1 & 2; MDOF = All Translations of Selected Nodes

Case (b) Measured Modes = 3 & 4; MDOF = All Translations of Selected Nodes

Figure 3. Parameter estimates for groups $I_{b1}, I_{b2}, I_{c}$. Modeling error in $K_{hh}, K_{vv}, K_{\theta\theta}$.

Case (a) Measured Modes = 1 & 2; MDOF = All Translations of Selected Nodes

Case (b) Measured Modes = 3 & 4; MDOF = All Translations of Selected Nodes

Figure 4. Parameter estimates for groups $K_{hh}, K_{vv}, K_{\theta\theta}$. Modeling error in $I_{b1}, I_{b2}, I_{c}$. 
\( o = \text{stiffness-based modal error function}; \ x = \text{flexibility-based modal error function} \)

Case (a) Measured Modes = 1 & 2; MDOF = All Translations of Selected Nodes

Case (b) Measured Modes = 3 & 4; MDOF = All Translations of Selected Nodes

**Figure 5.** Parameter estimates for groups \( I_{b1}, I_{b2}, I_c \).

Modeling error in the right foundation in \( K_{hh}, K_{vv}, K_{\theta \theta} \).

Case (a) Measured Modes = 1 & 2; MDOF = All Translations of Selected Nodes

Case (b) Measured Modes = 3 & 4; MDOF = All Translations of Selected Nodes

**Figure 6.** Parameter estimates for the left three groups of foundations in \( K_{hh}, K_{vv}, K_{\theta \theta} \).

Modeling error in the right foundation in \( K_{hh}, K_{vv}, K_{\theta \theta} \).
3.3 Remarks

There are three types of parameter estimate behavior: convergence, partial convergence, and divergence. There are some cases of partial convergence where one or more of the unknown parameters converged while the rest diverged. This behavior is shown in figures 4, 5(b), and 6(b). Only cases with either convergence or partial convergence were reported.

Using the stiffness-based modal error function; $K_{sv}$, was estimated with zero percentage error for both types of modeling error and both measurement scenarios. This performance for $K_{sv}$ estimation is superior to that of the flexibility-based modal function. For both error functions when mode shapes 3 and 4 are measured, foundation stiffness parameters $K_{hh}$ and $K_{hh}$ are estimated more accurately compared to using modes 1 and 2 in presence of either global or local modeling error as shown in figures 4 and 6. The middle beam $I_{b2}$ was estimated more accurately compared to the edge beams $I_{b1}$ and columns $I_c$ when using stiffness-based error function with either case of measured modes of vibration.

Generally, the stiffness-based modal error function was more robust and less sensitive to modeling error than flexibility-based error modal function when modeling error was introduced to the structure.

4. CONCLUSIONS

Parameter estimation has the potential to be a useful tool for model updating of in-service structures if influence of modeling error on the parameter estimates is controlled. As shown in the presented simulations modeling error can have a significant impact on parameter estimation, which can not be ignored when used for model updating. A simulated study of the impact of uncertainty in some structural stiffness parameters on the estimated values provides a tool for understanding the behavior of the model in anticipation of using field data. The sensitivity of the parameter estimates to modeling error is dependent on the type and magnitude of modeling error applied to the model, the uncertain parameters, the modal error function used in the parameter estimation process, the measured modes of vibration, and the location of the measurements. Both modal error functions used are unbiased meaning that with no modeling error, they are capable of obtaining exact parameter estimates. Overall, the 'stiffness-based modal error function' estimated the structural parameters more accurately than the 'flexibility-based modal error function' in presence of modeling error.

Future work should focus on NDT design to determine the optimal combination of measured modes of vibration at selected DOF for successful parameter estimation. The ideal combination should be robust in the presence of modeling error and measurement error.

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REFERENCES


PARIS®, PARameter Identification System. (1997), Department of Civil and Environmental Engineering, Tufts University, Medford, Massachusetts.


