Multiresponse Parameter Estimation for Finite-Element Model Updating Using Nondestructive Test Data

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Abstract: Structural health monitoring using field measurements has developed into a major research area, responding to an increasing demand for evaluating the integrity of civil engineering structures. Model updating through parameter estimation is a key tool in a successful structural health monitoring program. A method for parameter estimation is developed for simultaneous use of static and modal nondestructive test data called the “multiresponse” parameter estimation. An error function normalization technique is also developed to facilitate effective multiresponse parameter estimation. This normalization technique can mitigate some of the numerical issues encountered during the parameter estimation procedure. However, this technique does not degrade the integrity of the parameter estimation procedure. Multiresponse parameter estimation provides an increased level of flexibility and feasibility of model updating for structural health monitoring. This paper presents full integration of static and modal nondestructive test data using both stiffness-based and mass-based error functions for structural health monitoring. A benchmark laboratory grid model of a bridge deck is utilized to illustrate application of both normalization and multiresponse parameter estimation for updating the stiffness and mass parameters using nondestructive test data.

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Introduction

It is an undisputed fact that the asset management and maintenance of the United States infrastructure system presents a significant set of challenges to federal, state, and local government agencies. Highway bridges are a key component of the transportation infrastructure system. Of the approximately 590,000 highway bridges in the United States, 27% are considered structurally deficient or functionally obsolete (ASCE 2005). One major challenge is to find a cost effective maintenance system that provides useful information about the infrastructure in an efficient manner (Phares et al. 2000; and Aktan et al. 2000).

The current managerial focus for bridge systems requires the ability to plan and forecast levels of structural deterioration and the need for maintenance or rehabilitation procedures. Damage can accumulate during the life of a structure and reach a level such that the structure becomes deficient. Also, some forms of damage may remain unidentified due to the inability of visual methods to observe the damages and can lead to component failure or catastrophic failure in the absence of an effective structural health-monitoring program.

Structural parameter estimation is the art of reconciling an a priori finite-element model (FEM) of the structure with nondestructive test (NDT) data from the structure. Structural parameter estimation has a great potential for the purpose of finite-element model updating for structural health monitoring of in-service structures, specifically as part of a current bridge management system.

For finite-element based parameter estimation, the structure is first modeled with discrete elements assembled with known and unknown mass and stiffness properties and boundary conditions. The presented multiresponse parameter estimation procedure can systematically adjust both the mass and stiffness of the unknown parameters using nondestructive test data. Some examples of the stiffness properties are axial rigidity ($E_A$), flexural rigidity ($E_I$), torsional rigidity ($G_I$), support stiffness ($k$), lumped mass ($M$), and element distributed mass per unit length ($\rho$), where $E$ = modulus of elasticity; $G$ = shear modulus; $A$ = cross-sectional area; $I$ = moment of inertia; and $J$ = polar moment of inertia. The estimates of mass and stiffness parameters are determined during parameter estimation and then used to update the FEM. The difference between the “design” parameters and the estimated parameters reveal the condition change in the structure. Using a discrete mathematical model, the parameter estimates reveal not only damage location but also damage severity. Parameter estimation can also help determine the current load rating of an in-service bridge accounting for any loss in stiffness during the life of the bridge. It can also be used to predict the remaining life of in-service structures given current loading conditions. Fig. 1

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Parameter estimation is a more systematic and objective way of assessing damage in a structure. It is achieved by determining mass and stiffness properties of structural members in question that reflect the field measurements. The estimated sectional property values can then be compared to values from as-built structural drawings and the severity of damage can be assessed.

With recent advancement in computational capabilities, more advanced methods of both parameter estimation and measurement location selections have been developed. Advancements in these areas have increased the engineer’s ability to perform accurate damage identification and model updating, creating a more effective structural health monitoring program. Farrar et al. (2003) and Farrar and Jauregui (1998) summarize the current, state of the art, damage identification methods using measured modal responses. Generally, parameter estimation techniques compare the predicted analytical response of a structure with the actual measured response. Both Aktan et al. (1997) and Jang et al. (2002) offer a comprehensive study of the integration of the analytical and the experimental sides of parameter estimation. For this purpose, parameters of the FEM are updated until the predicted response is within prespecified bounds of the measured response. Although this procedure may seem relatively simple, there are several sources of error that can impede the practical application of parameter estimation.

Literature in the area of parameter estimation for health monitoring of structures is extensive. Stubbs and Osegueda (1990) develop a theory using changes in modal characteristics in beams, plates, and shells to detect damage. This method was refined and applied to offshore structures by Kim and Stubbs (2002). Yeo et al. (2000) propose a statistical approach to static parameter estimation through hypothesis testing. Hjelmstad and Shin (1997) developed a method suited to sparsely sampled static and modal responses used independently. Doebling et al. (1998) uses vibration test data to identify a structure’s local stiffness through the disassembly of the flexibility matrix. Frequency domain data, which are more compact than time domain data and more readily reveal the modes of vibrations, are used for parameter estimation in Koh et al. (2000). He and De Roeck (1997) present a method of structural damage detection using autoregressive modal data. Guo (2002) presents a method to improve a structural model using vibration test data and an element-level energy error estimation method to identify poorly modeled regions of stiffness and mass in a structure distinguishing these from the better modeled regions.

There are several optimization methods currently available for determination of the parameter estimates (Venkataraman and
Haftha et al. 2004). Arya and Sanayei (2000) integrated genetic algorithms (GAs) with traditional hill climbing (HC) methods to initially locate the vicinity of global minimum using GA and switching to HC for rapid convergence. Chou and Ghaboussi (2001) present the application of GA to structural damage detection using static measurements of displacements. Lee et al. (2005) present a neural network-based damage detection method. In this method, mode shape ratios are used as input to the neural network and serve to reduce the effect of the modeling error in the baseline FEM. Franco et al. (2004) present a parameter estimation technique based on an evolutionary strategy. The method shows promise with simulated modal data and avoids shortcomings of classical optimization methods such as the need for reliable initial estimates. This paper uses the Gauss–Newton and conjugate gradient methods (Chong and Zak 2001) for parameter estimation.

Parameter estimation can be heavily influenced by measurement noise (measurement error) and uncertainties in the known parameters (modeling error). A significant source of error that can drastically impact the validity of the results of parameter estimation is modeling error or the uncertainty in the parameters of a FEM. It is caused by incorrect assumptions used in the creation of the initial FEM. Examples of modeling error are an overestimation or underestimation of the initial stiffness and mass properties as well as changes due to subsequent deterioration or damage. Yeun et al. (2004) use a two-step process for structural parameter estimation of Phase I benchmark studies to mitigate uncertainty and bias error. For further information regarding impact of modeling errors on parameter estimation modeling error refer to Sanayei et al. (2001) and Chase et al. (2005).

The success of the parameter estimation process is also dependent on the quality, location, and type of NDT measurements. The sensor location and type of sensors (input) play a major role in the accuracy of the parameter estimates (output). Recent advancement in sensor technology has improved the accuracy of the measurements. With proper placement and data acquisition system, the amount of noise or measurement error in the preprocessed NDT data sets can be limited to low levels. The quality of the NDT measurements is controllable in terms of measurement techniques and measurement apparatus (Aktan et al. 1997; Sanayei et al. 1992).

This paper focuses on combining multiple data types for multiresponse parameter estimation of both stiffness and mass parameters. Other researchers (Wang et al. 2001 and Oh and Jung 1998) have worked on combining static and modal data on a limited scale. The proposed method uses a modified error function protocol for statistical parameter estimation. Kiddy and Pines (1998) present a technique for simultaneous updating of mass and stiffness matrices using a sensitivity-based method. They propose a constraint on the number of unknown parameters to avoid the problem of simultaneous updating multiple parameters. Cathas et al. (2004) investigated the use of multiple inputs for modal analysis of large-scale structures. The procedure presented in this paper allows for elemental structural properties influencing system stiffness to be estimated independently from the properties that influence system mass (Javdekar 2004).

### Parameter Estimation

A Matlab-based parameter estimation program, PARmeter Identification System (PARIS) (Sanayei 1997), developed at Tufts University, has been utilized to estimate the unknown stiffness and mass parameters of the structural elements of a FEM. Typically, structural parameters for finite-element model updating include axial rigidity (EA), flexural rigidity (EI), and torsional rigidity (GJ), foundation stiffness (k), mass per unit length (m = ρAa), and lumped mass (M).

The structure can be excited either statically with applied loads, F, measuring displacements and rotations, U, or dynamically measuring frequency response functions of lightly damped systems and extracting natural resonance frequencies, ωi, and associated mode shapes, Φi, for linear parameter estimation. A selected number of measurements gathered sparsely at certain strategically selected degrees of freedom (DOF) can be used for parameter estimation. PARIS can utilize both static and modal data such as displacements and rotations under stationary loads, or extracted natural frequencies and associated mode shapes for parameter estimation. PARIS can simulate and also receive both complete and sparse static and modal data sets.

Four distinct error functions are used for multiresponse parameter estimation in this paper. However, the combining technique presented is applicable to several other error functions within PARIS such as those based on strain measurements (Sanayei et al. 1997) among others. A brief formulation of each error function used in this paper is presented here for clarity in the example.

PARIS also allows for grouping the several parameters that have same stiffness and mass properties and also are expected to have similar final estimated values together. Grouping allows for a smaller total number of unknown parameters and consequently a smaller number of required measurements for estimating those parameters, both of which increase the computational speed. The unknown parameter grouping schemes can be refined based on the change in the grouped parameter estimates. A significant change in a grouped parameter estimate indicates that one or more of the parameters in that group have experienced damage. It is feasible to methodically subdivide the unknown parameter group to locate the damaged member.

### Static Stiffness-Based Error Function

The static stiffness-based error function, \( E_{sS}(p) \), was developed by Sanayei and Nelson (1986)

\[
E_{sS}(p) = \| K(p) \| U - F = [F_{predicted}] - [F_{measured}] \tag{1}
\]

This error function is based on the residual forces at a subset of DOF. It is essentially the difference between the predicted and measured forces. \( [F_{predicted}] \) is calculated using the analytical stiffness matrix and the set of measured displacement, \( [U] \), from an NDT at a subset of DOF. \( [F_{measured}] \) = applied set of live load cases. The complete formulation using a subset of measurements that can be different from the applied load is presented by Sanayei and Onipede (1991).

### Static Flexibility-Based Error Function

The static flexibility-based error function, \( E_{sf}(p) \), was developed by Sanayei et al. (1997)

\[
E_{sf}(p) = \| K(p)^{-1} F - U \| = [U_{predicted}] - [U_{measured}] \tag{2}
\]

This error function is based on the residual displacements at a subset of DOF. Similar to the formulation of \( E_{sS}(p) \), \( [U_{predicted}] \) is calculated using the analytical stiffness matrix and the set of applied live load cases. The measured displacement set, \( [U_{measured}] \)
Modal Stiffness- and Mass-Based Error Function

Using the basic modal analysis theory, the modal stiffness-based error function, \( E_{ms}(p) \), was developed by Gornshteyn (1992) and enhanced by Sanayei et al. (1999)

\[
E_{ms}(p)_{ij} = [K(p)](\Phi)_{ij} - \omega^2_i[M(p)](\Phi)_{ij}
\]

(3)

This error function is based on the residual modal elastic and inertia forces predicted at a subset of DOF. It uses the mode shapes, \( \{\Phi\} \), and natural frequencies, \( \{\omega\} \), extracted from an NDT data set. The natural mode of vibration used is labeled with subscript \( j \), \( K(p) \) and \( M(p) \) = analytical stiffness and mass matrices.

Modal Flexibility- and Mass-Based Error Function

The modal flexibility-based error function, \( E_{mf}(p) \), was developed by Arya (2000) and enhanced by Sanayei et al. (2001)

\[
E_{mf}(p)_{ij} = \omega^2_i[K(p)^{-1}][M(p)](\Phi)_{ij} - \Phi_{ij}
\]

(4)

Similar to the formulation of \( E_{ms}(p) \), this error function is based on residual modal displacements predicted at a subset of DOF.

Minimization of the above error functions leads the search for parameter estimation of the objective function, \( J(p) \), that is the Frobenius norm of any error function

\[
J(p) = \sum_i \sum_j E(p)_{ij}^2
\]

(5)

where \( i = \text{measured DOF} \); and \( j = \text{measured load case or mode of vibration} \).

Error Function Normalization in Multiresponse Parameter Estimation

When working with multiple error functions, some numerical difficulties are encountered. A wide range of numerical values is processed when using multiple sets of measurement types due to usage of different units and scale. For example, displacements are measured in “inches,” rotations are measured in “radians,” and strains are measured in “\( \mu \text{inch/inch} \)” in multiresponse parameter estimation, it is essential that one measurement type does not overshadow another, therefore error function normalization (EFN) is required.

When normalizing the error function, the parameter estimation process is manipulated at its core. The error function (residual) is the basis for the parameter estimation procedure. The disparity between cell values in the error matrix can magnify the peaks and valleys that are inherent to the objective function surface (cost function), complicating the parameter estimation. The role of EFN is to smooth the objective function surface without muting the actual global minimum. Normalization is applied both at the parameter level (Sanayei et al. 1999) and at the error function level (Santini-Bell and Sanayei 2005).

In addition to smoothing the variation in magnitudes within the \( E(p) \), EFN also prepares the different error functions to be used simultaneously. For example, the units associated with the \( E_{mf}(p) \) are forces and moments while \( E_{ms}(p) \) has units of displacement and rotations. Using EFN would make both \( E_{mf}(p) \) and \( E_{ms}(p) \) untensile and decrease the chance that one error function would overshadow the other (Santini 2003). This paper uses the initial value of the error functions \( E_{mf}(p) \) calculated using initial estimates of parameters for normalizing the error functions. Typically, information from existing as-built structural drawings and field measurements is the basis for the calculation of the initial parameter values.

The EFN based on initial value, \( E_{mf}(p) \), uses the a priori parameter values, \( \{p\} \) for creating an error function matrix \( E_{mf}(p) \). Each entry of this matrix can have contributions from several unknown parameters. The error function \( E(p) \) and the sensitivity matrix \( S(p) \), which is the partial derivative of the error function with respect to each unknown parameter, in each iteration, including the first iteration, is divided by element by element by \( E_{mf}(p) \) for normalization at the matrix cell level. The EFN must be applied to both the \( E(p) \) and \( S(p) \), since \( S(p) \) is what directs the search for the global minimum using both the Gauss–Newton and conjugate gradient optimization methods presented in this paper

\[
E_{sf}(p) = \frac{[E(p)]}{[E_{mf}(p)]}
\]

(6)

This type of normalization creates both a unitless error function and a unitless sensitivity matrix. Matrices \( [E(p)] \), \( [S(p)] \), and \( [E_{mf}(p)] \) are of the same sizes \( \text{NMDOF} \times \text{NSF} \) (number of measured DOF \times number of sets of forces or natural modes). \( \{p\} \) = vector of unknown parameters and \( \{p\} \) represents its initial values of size \( \text{NUP} \times 1 \) (number of unknown parameters). By using EFN, each component that contributes to the error function matrix is affected. In order to account for different sensors, PARIS has the ability to apply weight factors. Because this was a laboratory experiment under controlled circumstances, the weighting option was not used for this scenario.

Multiresponse Parameter Estimation

When different error functions that are based on different measurement types are used it is necessary to combine such data in a systemic and appropriate fashion to ensure that the integrity of the data is not compromised. A stacking procedure can be applied to the combination of load cases or modes of vibration using different error functions. The stacking allows the use of several different measurement types simultaneously, increasing the number of unknown parameters that can be estimated and allowing the combination of the static and modal data.

Within each active normalized error function \( E(p) \), a vector \( \{E(p)\} \) of size \( \text{NM} \times 1 \) is calculated where \( \text{NM}=\text{NMDOF} \times \text{NSF} \). Then each \( E(p) \) is stacked to create a \( E_{\text{stack}}(p) \) which contains the information from each active error function for the entire multiresponse parameter estimation scenario.
The UCII grid was a 3.65 m (12 ft) × 1.83 m (6 ft) grid providing enough laboratory space for the required monitoring and data collection equipment. The UCII grid members were 7.62 cm (3 in.) × 5.08 cm (2 in.) × 0.476 cm (3/16 in.) structural steel tubing, in both the transverse and longitudinal directions. Fig. 3 represents the drawing of the UCII grid’s final design. The connection zones between the tubing were constructed using 0.476 cm (3/16 in.) steel plates with A307 0.635 cm (1/4 in.) diameter bolts. Detailed drawings for each type of connection are shown in Fig. 4. To simulate the performance of a bridge deck, four neoprene pads supported the UCII grid on the sawhorses at the four corners as shown in Fig. 5.

**Type and Method of Nondestructive Testing on UCII Grid**

Both static and modal NDT were performed on the UCII grid using static loads at predetermined locations on the UCII grid and using an impact hammer. Three identical static NDT were performed on the UCII grid using 222 N (50 lb), 444 N (100 lb), and 666 N (150 lb) loadings. For the static tests, vertical displacements, rotations, and strains were measured. The measurement locations for each date type were the same for all three tests (see Fig. 6). Based on the data quality analysis performed by Northeastern University (Wadia-Fascetti et al. 1999) only data from Test 3 were used for parameter estimation.

Experimental modal analysis was used to determine the modal parameters (frequencies and mode shapes) of the assembled UCII grid. An impact hammer was used to excite all 21 nodes and accelerations were measured with all 21 accelerometers. Next, the measured data were processed at UCII using frequency and time domain techniques to obtain the modal shapes and natural frequencies of the structure.

**Static NDT Displacement Data Selection**

The analysis of the data revealed that many of the load cases, where the load was located toward the edges of the UCII grid,
created uplift at some locations along the opposite edges. Since the FEM of the UCII grid could not accurately account for the uplift, these load cases were discarded. Each static loading and measurements was repeated ten times. Basic statistical analysis on the ten sets of measurements was performed and those measurements with high standard deviations were discarded. For consistency, when a measurement was discarded, the entire load case was also discarded to maintain a constant 11 static displacement measurements for each load case used (Fig. 6). As a result of these elimination techniques, three 666 N (150 lb) load cases remained to be used as data for parameter estimation. These three independent load cases were located at E3, G3, and I3; the middle loading points along Line 3. The three load cases were applied at the UCII grid connections individually and yielded a total of 33 vertical displacement measurements.

Dynamic NDT Measurement Location Selection

Modal identification was performed using data from time domain (TD) and frequency domain (FD) tests, which was in the form of frequency response function (FRF), was postprocessed, and natural frequencies and mode shapes were extracted. The modal parameters acquired from the TD and FD tests were then compared to confirm their consistency by Ciloglu et al. (2001) and Slavsky (2005). The FD tests generated a repeating frequency for Modes 4 and 5. In contrast, TD tests generated distinct frequencies however, with some degree of modal coupling. The resulting mode shapes from TD Test 3 are shown in Fig. 7. The TD modal data were used for the parameter estimation presented.

Parameter Estimation for UCII Grid Experiment

The first step in parameter estimation is to create a FEM that can capture the general behavior of the structure. The FEM of the
UCII grid uses separate elements for the connection zones and structural tubes. Parameter groupings are used to reduce the number of unknown parameters for similar structural elements. By changing the grouping configuration of the unknown parameters, rather than changing the elements in the FEM itself, different parameter estimation scenarios were created for the UCII grid.

Given the purely vertical nature of both the static loadings and modal excitations, the measured NDT data indicate that there is no horizontal motion (in the X and Y direction) or twisting motion (about the Z axis). In order to both simplify the FEM, in terms of the number of DOF, and create a FEM that would best reflect the NDT data using a smaller number of DOF, $U_X$, $U_Y$, and $\theta_Z$ DOF of the FEM were restrained. Based on the nature of the load cases and excitations, only vertical displacements were expected for the UCII grid. Static displacements and modes of vibration from both the unrestrained and restrained FEM were compared to ensure that there were no adverse affects caused by the restrained DOF.

### Finite-Element Model for UCII Grid

The FEM for the UCII grid was created with 85 nodes, 96 frame elements, and four linear spring elements. This FEM was created in both PARIS and SAP2000. Three-dimensional beam elements were used to represent both the beams and the connection zones. For the support pads, linear springs with stiffness only in the translation directions, $U_X$, $U_Y$, and $U_Z$, ($K_{UX}$, $K_{UY}$, and $K_{UZ}$) were used.

In order to account for the additional weight of the bolts, the clip angles, gusset plates, and sensors or the loss of weight due to drilled holes, the cross-sectional area $A_m$ represents the area-

### Parameter Estimation for Model Updating Using NDT Data

For the UCII grid model updating, two model cases were created. Model Case 1 estimated individual bearing pad stiffnesses and two grouped connection stiffness and mass parameters. Model Case 2 used the pad stiffness estimates and refined the connections to four groups of stiffness and mass parameters (Slavsky 2005). In both model cases, the TS section properties were considered known. In Model Case 1, the two categories of unknown parameters were: (1) the bearing pads and (2) the connection zones. The four bearing pads have a major impact on the overall response of the UCII grid, and as a result the vertical spring stiffness parameters for the bearing pads were not grouped and estimated individually. Since there are a large number of connection elements, those that were expected to behave similarly were grouped together to a single unknown parameter which reduces the total number of unknown stiffness and mass parameters. For Model Case 1, the exterior connections were grouped together and interior connections to a second group.

Eight parameters were used in Model Case 1: $K_1$, $K_2$, $K_3$, $K_4$, $I_1$, $I_2$, $A_{m1}$, and $A_{m2}$. Connections were grouped into “external” (Parameters $I_1$ and $A_{m1}$) and “internal” (Parameters $I_2$ and $A_{m2}$). Parameter estimation was performed using only static test data to gain a better understanding of the differences in the FEM predicted responses versus and the measured responses. It revealed that the majority of the static stiffness changes occur in the four corner pad stiffness values. The estimates of the vertical stiffness

### Table 1. Initial Section Properties of the Structural Members

<table>
<thead>
<tr>
<th>Structural member type</th>
<th>$A$ [in.$^2$ (cm$^2$)]</th>
<th>$A_m$ [in.$^2$ (cm$^2$)]</th>
<th>$I_{xx}$ [in.$^4$ (cm$^4$)]</th>
<th>$I_{yy}$ [in.$^4$ (cm$^4$)]</th>
<th>$I_{zz}$ [in.$^4$ (cm$^4$)]</th>
<th>$J$ [in.$^4$ (cm$^4$)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tubes (TS 3 × 2 × 3/16)</td>
<td>1.64 (10.6)</td>
<td>1.64 (10.6)</td>
<td>1.86 (77.4)</td>
<td>0.977 (40.7)</td>
<td>2.16 (89.9)</td>
<td></td>
</tr>
<tr>
<td>Connection elements</td>
<td>1.64 (10.6)</td>
<td>3.70 (23.9)</td>
<td>1.86 (77.4)</td>
<td>7.73 (321.7)</td>
<td>2.20 (91.6)</td>
<td></td>
</tr>
</tbody>
</table>
of the bearing pads are: $K_1 = 12.66$ kN/cm (7.23 kip/in.), $K_2 = 14.8$ kN/cm (8.43 kip/in.), $K_3 = 13.1$ kN/cm (7.50 kip/in.), and $K_4 = 13.4$ kN/cm (7.65 kip/in.) (Slavsky 2005). The FEM was updated using the vertical stiffness values of the springs.

In Model Case 1, the connection elements were subdivided into two groups (interior and exterior). From a comparison of displacements at bearing pads with NDT data, it was clear that a displacement match with NDT data was achieved. Since the longitudinal elements of the UCII grid was observed to play a major role in the load transfer and controlling the displacements, three connection groups were created for the longitudinal elements of the UCII grid and the transverse connection elements were grouped into a single group for Modal Case 2.

Fig. 8 illustrates how the 64 connection elements logically and effectively are separated into four meaningful groups for parameter estimation of Model Case 2. Group 1 represents the 28 cross (transverse) connection elements grouped as one. Groups 2, 3, and 4 each represent 12 longitudinal connection elements of Lines 1, 3, and 5, respectively. Each group represents one unknown stiffness and one unknown mass parameter each creating eight unknown parameters to be estimated using multiresponse parameter estimation.

Multiresponse parameter estimation requires EFN. A comparison of the performance of the normalization methods presented in this paper was completed using both the Gauss–Newton and conjugate gradient optimization techniques. This comparison shows the very effective performance of EFN. All multiresponse parameter estimation cases presented will use normalization with respect to the initial parameter values, $[E_0(p)]$.

**Multiresponse Parameter Estimation Results**

The scope of this parameter estimation is to update both mass and stiffness properties of the connection elements, $4I + 4A_m$. As discussed above, the four pad stiffness values are considered known due to their successful estimation using only the static data leading to a close match of displacements at the four corner pads with NDT data (Model Case 1). Therefore, the unknown parameter groups are the moment of inertia and mass parameter of the four connection groups leading to a total of eight unknown parameters (Model Case 2). The surface plots, Fig. 9, show the complexity of the flexibility-based error surfaces. The right graphs in Fig. 9 show the path of the parameter estimation from the initial value to the final parameter estimates. The parameter estimates from the flexibility-based cases are more reflective of the laboratory NDT data, as shown in Figs. 10 and 11, therefore they are the only surfaces that are presented.

Visualization of the objective function surfaces plots ($J$ plots) is a powerful method to gain insight to the nature of the individual parameter estimation scenario and optimization process. For plotting the objective function $J(p)$ for “$n$” unknown parameters, the surface will be in $n + 1$ dimensional space. Since it is not possible to plot in more than the three dimensions (3D), the surface plots in 3D are created using only two unknown parameters. The rest of the unknown parameters are maintained at their final estimated parameter values. This is analogous to slicing a multi-dimensional space to observe the objective function surface versus only two unknown parameters.

The flat surface of the error surface shown in Fig. 9 indicates that this parameter estimation scenario is suited for the conjugate gradient optimization technique. Due to the relatively large step size of the Gauss–Newton technique, surfaces with many local minima and a flat surface in the vicinity of the global minimum are better served by the conjugate gradient due to the small step nature of the algorithm (Slavsky 2005).

For Model Case 2, the moment of inertia and area mass parameter estimate values of both the stiffness-based and flexibility-based case are summarized in Table 2. The flexibility-based case results indicate the highest parameter estimates occur in the UCII grid Line 3 group ($I_3$ and $A_{m3}$). The second highest area masses occur in Lines 1 and 5, and the lowest area mass values occur in the area mass of the cross/transverse connection elements ($A_{m1}$). These results make sense physically, since the largest gusset plates with the most bolts are used along the middle line (Line 3).

The parameter estimates from the stiffness-based case do not reflect visual observations of the UCII grid and engineering judgment based on the known construction techniques of the UCII grid. This is an early indication that these parameter estimates do not reflect the condition of the UCII grid. This observation is proved accurate when the response from the updated model using the parameter estimates from both the stiffness and flexibility based error functions are compared with the NDT data in Figs. 10 and 11.

It is important to assess the physical meaning of parameter estimates derived from the stiffness and flexibility-based cases. Even though inspection of the numerical results reveal significant information regarding the fitness of the parameter estimation, direct comparison with the NDT data provides an objective and quantifiable basis for comparison. Each set of parameter estimates for the benchmark UCII grid were used to update the FEM. Both the static and modal responses for the updated models were then compared with the NDT data. The tools of simulated displacement plots, frequency matching, and modal assurance criterion (MAC) matrices are used to compare the predicted response to the NDT data. Based on the comparisons detailed below, the flexibility based $4I + 4A_m$ case is considered the best multiresponse case.

Displacement plots for each of the three longitudinal UCII grid lines were created using the estimated parameter values, shown in Table 2, and the measured NDT data. All three plots show excellent match between the NDT measurements and updated model response for the flexibility-based case and not such a good match for its stiffness-based counterpart (Fig. 10). It is important to remember that static displacements are impacted by the stiffness parameters alone and not by the mass parameters.
Fig. 9. Objective function surface plots and convergence paths for $4I + 4A_m$, SF+MF case, conjugate gradient
Both stiffness and mass parameters affect modal frequencies and mode shapes of a structure. A visual chart of NDT versus predicated frequencies for both stiffness based and flexibility based cases is shown in Fig. 11. It can be seen in the chart that the flexibility based case produces a closer match to the NDT frequencies. Table 3 shows the MAC values measuring the degree of fitness between the mode shapes extracted during NDT and the corresponding mode shapes obtained from (a) the initial FEM and (b) the updated FEM. Note that the NDT modes are shown in the vertical column and the FEM modes are shown in the horizontal row.

Using results of Model Case 2, Table 3(a) indicates a strong coupling of Modes 3 and 4 and also problems with Modes 7 and 8 being out of order. This was discovered by comparing the mode shapes from the analytical model using the initial parameter values with the experimentally identified mode shapes. As a result only Modes 1, 2, 3, 6, and 9 were used for parameter estimation measured by accelerometers at 21 vertical DOF (all 21 connections). Comparison of the MAC values indicate improvement in MAC values (closer to 1.0) for measured Modes 1, 2, 3 6, and 9. Based on a comparison with both static and modal NDT data, the flexibility-based $4I+4A_m$ case is considered the best multisresponse parameter estimation case. Fig. 11 shows that the correlation between the modal frequencies has significantly improved.
Fig. 11. Frequency comparison chart for 4I+4Am scenario

Table 2. Parameter Estimate Values from Multiresponse, 4I+4Am Scenario

<table>
<thead>
<tr>
<th>Parameter Estimate</th>
<th>Initial parameter values</th>
<th>4I+4M, SS+MS</th>
<th>4I+4M, SF+MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1$-Cross [cm$^4$(in.$^2$)]</td>
<td>77.4 (1.86)</td>
<td>72.8 (1.75)</td>
<td>129.9 (3.12)</td>
</tr>
<tr>
<td>$I_2$-Line 3 [cm$^4$(in.$^2$)]</td>
<td>77.4 (1.86)</td>
<td>65.8 (1.58)</td>
<td>106.6 (2.56)</td>
</tr>
<tr>
<td>$I_3$-Line 3 [cm$^4$(in.$^2$)]</td>
<td>77.4 (1.86)</td>
<td>62.9 (1.51)</td>
<td>124.9 (3.00)</td>
</tr>
<tr>
<td>$I_4$-Line 5 [cm$^4$(in.$^2$)]</td>
<td>77.4 (1.86)</td>
<td>64.9 (1.56)</td>
<td>74.5 (1.79)</td>
</tr>
<tr>
<td>$A_{m_1}$-Cross [cm$^2$(in.$^2$)]</td>
<td>23.9 (3.70)</td>
<td>8.9 (1.38)</td>
<td>15.5 (2.38)</td>
</tr>
<tr>
<td>$A_{m_2}$-Line 1 [cm$^2$(in.$^2$)]</td>
<td>23.9 (3.70)</td>
<td>26.2 (4.06)</td>
<td>24.9 (3.87)</td>
</tr>
<tr>
<td>$A_{m_3}$-Line 3 [cm$^2$(in.$^2$)]</td>
<td>23.9 (3.70)</td>
<td>28.5 (4.41)</td>
<td>30.9 (4.80)</td>
</tr>
<tr>
<td>$A_{m_4}$-Line 5 [cm$^2$(in.$^2$)]</td>
<td>23.9 (3.70)</td>
<td>24.8 (3.84)</td>
<td>22.9 (3.56)</td>
</tr>
</tbody>
</table>

No. of iterations for convergence

Table 3. MAC Values (a) Using Initial Parameter Value; (b) Using 4I+4Am, SF+MF Case

<table>
<thead>
<tr>
<th>Modes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(a) MAC values using initial parameter values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.990</td>
<td>0.000</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.062</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.964</td>
<td>0.005</td>
<td>0.002</td>
<td>0.002</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.006</td>
<td>0.907</td>
<td>0.014</td>
<td>0.017</td>
<td>0.007</td>
<td>0.004</td>
<td>0.000</td>
<td>0.032</td>
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<tr>
<td>4</td>
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<td>0.001</td>
<td>0.072</td>
<td>0.472</td>
<td>0.371</td>
<td>0.024</td>
<td>0.020</td>
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<td>0.013</td>
<td>0.003</td>
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<td>0.544</td>
<td>0.001</td>
<td>0.009</td>
<td>0.033</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.048</td>
<td>0.000</td>
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</table>

(b) MAC value using parameter values of case 4I+4Am, SF+MF

<table>
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<tr>
<th>Modes</th>
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<th>4</th>
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<th>6</th>
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<tr>
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<td>0.990</td>
<td>0.007</td>
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<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
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<tr>
<td>3</td>
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<td>0.001</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.007</td>
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<td>0.000</td>
<td>0.008</td>
<td>0.008</td>
<td>0.000</td>
<td>0.967</td>
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</table>

Fig. 10 illustrates that the static vertical displacements are significantly closer to the NDT data as compared with that of the stiffness-based case. The superiority of the flexibility-based error function parameter estimates as compared to the results from the stiffness-based error function is unique to this structure and the associated parameter estimation scenario.

Conclusions

This research demonstrates that it is feasible to simultaneously estimate stiffness and mass parameters using multiresponse parameter estimation with static and modal NDT data. The necessity and effectiveness of error function normalization with multiresponse parameter estimation due to varying scale were presented in this paper and illustrated in the benchmark UCII grid laboratory model. The importance of mode shape behavior matching was demonstrated with respect to the results from mass-based...
parameter estimation. The objective function surface plots and convergence path plots were both used as a means to assess the behavior of the optimization algorithms used in parameter estimation. These plots provide a visual insight and serve to unlock some of the black-box feeling associated with parameter estimation in a multidimensional parameter space.

Since static displacements and modal data were used simultaneously there is more NDT data available for parameter estimation. As a result, more structural parameters can be estimated per set of field measurements. This paper illustrated that multiple data types can be successfully combined to perform parameter estimation. A larger and more assorted data set can provide flexibility to the experimenters when performing data quality analysis and selection of the subset of measurements. Two model cases were used for parameter estimation and model updating. Since the UCII grid bearing pads were much more flexible than the grid, only static data were used to identify the four bearing pad stiffnesses in Model Case 1. Using this updated model a match at the bearing pad locations was achieved between the predicted and measured responses. Model Case 2 used a combination of static and model NDT data for estimation of stiffness and mass parameters of the connections. Although only eight parameters were estimated in Model Case 2, those eight parameters represented 49 structural parameters, condensed through grouping. The parameter estimates using the flexibility-based error functions resulted in a better correlation between the updated model responses and laboratory NDT data for the UCII grid.

There are several examples of bridges that are heavily instrumented for structural health monitoring (Cuelho et al. 2006). In most cases this instrumentation is a combination of strain gauges and tiltmeters (Riad et al. 2006). However, the common problem that faces bridge owners and decision makers is how to interpret this data into a decision making, assessment management tool. Multiresponse parameter estimation, as presented in this paper, is a postprocessing tool that can use field measurements to obtain meaningful locations of stiffness and mass changes of the tested structures. This information can then be used to allocate funds and manpower for further visual inspection and rehabilitation if required. In closing, regardless of the mathematical tools available for model updating, the engineering judgment of the structural engineer is paramount to successful parameter estimation for meaningful model updating and structural condition assessment.

Acknowledgments

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