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A global water supply reservoir yield model with uncertainty analysis

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Received 29 January 2014, revised 12 August 2014
Accepted for publication 13 August 2014
Published 9 September 2014

Abstract
Understanding the reliability and uncertainty associated with water supply yields derived from surface water reservoirs is central for planning purposes. Using a global dataset of monthly river discharge, we introduce a generalized model for estimating the mean and variance of water supply yield, \(Y\), expected from a reservoir for a prespecified reliability, \(R\), and storage capacity, \(S\) assuming a flow record of length \(n\). The generalized storage–reliability–yield (SRY) relationships reported here have numerous water resource applications ranging from preliminary water supply investigations, to economic and climate change impact assessments. An example indicates how our generalized SRY relationship can be combined with a hydroclimatic model to determine the impact of climate change on surface reservoir water supply yields. We also document that the variability of estimates of water supply yield are invariant to characteristics of the reservoir system, including its storage capacity and reliability. Standardized metrics of the variability of water supply yields are shown to depend only on the sample size of the inflows and the statistical characteristics of the inflow series.

Keywords: reservoirs, yields, uncertainty, global, planning, water, climate change

1. Introduction

Without a surface water reservoir system, surface water supplies must be drawn directly from the river. Such systems have water supply yield reliabilities which depend solely on the natural variability of the river and thus, such systems are subject to long periods of drought when river discharges fall below expected yields. Surface water reservoirs are designed to increase both the water supply yield and its associated reliability, while simultaneously providing downstream ecological flow releases among other benefits. Surface water reservoirs have been constructed all over the world for controlling water supply variability and plans are still underway to develop more reservoirs especially in developing countries. Due to natural variability of streamflows which is expected to increase due to climate change and variability, combined with the often short streamflow records available for the design of reservoirs, it remains unclear how well such reservoirs will be able to ensure the delivery of prespecified water supply yields with the desired reliabilities. Efforts to control the variability of streamflow by constructing reservoirs will only be met with success if engineers, planners and managers understand the resulting variability of water supply yields and reliabilities, which is the topic of this study.

In the design of water supply reservoirs, the storage–reliability–yield (SRY) relationship is the tool that has traditionally been used to determine the size of the storage reservoir required for delivery of a water supply yield with a given reliability or the water supply yield that can be supplied from an existing or proposed reservoir with known storage capacity. Behavior analysis (BA) is the simulation method that has been widely used to develop such an SRY relationship (see McMahon and Adeloye\(^2\) 2005, p 86). Using BA, the minimum reservoir storage capacity required for delivery of a specified yield with a given reliability is determined by trial and error. For a given water supply yield and reliability of the reservoir, an initial estimate of the reservoir capacity, \(S\), and the initial water content of the reservoir is assumed.
of the complete historical streamflow record (or synthetically generated streamflows) while accounting for all the necessary outflows which may include: water supply, evaporation, seepage losses, minimum downstream releases and other operations is then accomplished using a reservoir simulation model. The reliability of the yield delivered by that reservoir is then estimated. If the reliability is unsatisfactory, \( S \) is adjusted iteratively until the chosen \( S \) meets the required reliability for a given yield. A variant of the BA approach that has been widely used in the USA and elsewhere is the mass curve (Rippl 1883) or its automated equivalent sequent peak algorithm (SPA) introduced by Thomas and Burden (1963). The SPA assumes failure free (100% reliable) reservoir operations over a prespecified planning horizon which is often based on a historical record of inflows. During future planning periods, inflows into the reservoir are likely to be wetter or drier than the historical record indicates, thus the actual reliability of reservoirs designed using the SPA approach remain unknown.

Use of simulation procedures to derive the steady-state SRY relationship is computationally intensive because a stochastic streamflow model and a reservoir simulation model must be combined and implemented repeatedly, using thousands of Monte Carlo experiments. Attempts have been made to develop generalized SRY relations that can be used to mimic the results of such Monte Carlo experiments based on the use of a stochastic streamflow models combined with a simple reservoir simulation model. The generalized SRY relationships introduced previously were designed to estimate the reservoir storage capacity \( S \). Such SRY relations are based on the hydrologic characteristic of the inflows into the reservoir, prespecified water supply yield and reliability of the reservoir yields. This study introduces SRY relationships suited for estimation of the water supply yield \( Y \), given a prespecified storage capacity and reliability.

Understanding the variability of reservoir water supply yield estimates is fundamental for water resource planners in evaluating the ability of reservoirs to protect against future droughts. Here we introduce a generalized global SRY model for estimating reservoir yields and we further employ that model to document the variability of the water supply reservoir yield estimates for a wide class of reservoir systems. In addition we illustrate how the resulting yield model can be used to evaluate the sensitivity of water supply reservoir yields to potential future climate change.

2. Literature review

Several studies have developed generalized SRY relationship for reservoirs fed by synthetic inflows generated from a stochastic streamflow model combined with a routing method (see Pegram 1980, Vogel and Stedinger 1987, Bayazit and Bulu 1991, Phien 1993, Bayazit and Önoz 2000, among others). Such ‘theoretical’ generalized SRY models are limited for use with systems where the stochastic nature of the annual streamflows is well approximated by the particular theoretical model assumed during the development of the SRY model. Fewer studies have used actual streamflows to develop such generalized SRY models as is the case here, such as Adeloye et al. (2003), McMahon et al. (2007b), Adeloye (2009a, 2009b), and Silva and Portela (2012). However, a review of literature by Kuria (2014) revealed that none of the existing generalized SRY relationship are suited for estimating the yields expected from a water supply storage reservoir. For example, solving the generalized SRY relationship introduced by McMahon et al. (2007b) for water supply yield, often results in yield ratios (yields divided by the mean of the streamflows) that are greater than unity. Yield ratios greater than unity are simply not feasible in practice. One of our goals is to develop a generalized SRY relationship that can be used to estimate water supply yields using the global database of monthly streamflow observations employed by McMahon et al. (2007b).

In the design of surface water reservoirs using SRY relationships, the yield is often assumed constant. However actual water supply yields are random variables. This is due to natural hydrologic variability combined with our limited knowledge of that hydrologic variability (sampling variability) resulting from the limited lengths of streamflow sequences available for applying SRY relationships in practice. In addition, future inflows into the reservoir are likely to differ significantly (i.e. to be wetter or drier) from the historical flows. Thus one can expect that estimates of storage, yield and/or reliability are all random variables which are subject to considerable variability due to limited knowledge of future hydrologic conditions which govern the SRY relationship. Considering the importance of water supply planning in the context of surface water reservoirs, remarkably few studies have attempted to document the sampling variability of storage, yield and reliability estimates of reservoirs. We could only find a few such studies by Phatarford (1977), Klemes (1979), Vogel and Hellstrom (1988) and Vogel and Stedinger (1988), and yet, none of those studies considered the uncertainty in estimates of water supply yield. Instead those studies evaluated the instability of SRY relationship of specific systems focusing on the variability of estimates of storage and reliability. What is missing from the existing literature is a rigorous and general evaluation of water supply yields for a wide range of reservoir systems subject to the type of hydrologic variations and conditions expected anywhere in the world. Our goal in this paper is to document the variability of water supply yield estimates.

Generalized SRY relationships developed here and by others are approximate, and are not intended to replace, the more computationally intensive and complex sequential simulation approaches used in reservoir feasibility, design and operations studies. Instead, they are more suitable for a range of preliminary water supply management and planning purposes described below.

Generalized SRY relations provide water resource planners and managers with a useful tool for improving their understanding of the general behavior of relationships among storage capacity, reliability and the water supply yield of reservoirs. Since generalized SRY relationships can represent the behavior of an extremely wide range of reservoir systems
Figure 1. A comparison of actual water supply yields (m$^3$ yr$^{-1}$) to the yields predicted from equation (1) for the 12 413 cases considered in the development of the model.

they have been used for numerous other purposes which we summarize here, and in addition we discuss some new applications which have not yet been attempted. Vogel and Stedinger (1988) used generalized SRY relations developed by Vogel and Stedinger (1987) to illustrate the value of stochastic streamflow models in the design of water supply reservoirs and to document the variability of estimates of reservoir storage capacity based on short streamflow records. Vogel et al (1999b) use generalized SRY relationships to explore the behavior of thousands of actual storage reservoir systems across the continental United States. Vogel et al (1997, 1999b, 2001), Lane et al (1999) and Brown et al (2012) used generalized SRY relationships to explore the impact of climate change on reservoir system performance. Many other applications of generalized SRY relationships are possible which have not yet been attempted. For example, SRY relationships could be used to determine the optimal storage capacity of a reservoir subject to particular constraints on water supply yield, instream flow requirements and reliability. Similarly, generalized SRY relationships could be used for the regional economic assessment of adding reservoir storage infrastructure to a region for the purposes of providing a more secure and stable water supply yield under future projections of water supply demands combined with additional uncertainties arising from potential climate change.

3. A global SRY relationship for water supply reservoirs

We used the same global dataset that was used by McMahon et al (2007b) which consisted of monthly streamflow series from 729 unregulated rivers all of which had at least 25 years of monthly streamflow data (see their figure 1 for the spatial coverage of the dataset). Of critical importance to the use of any empirically derived SRY relationship are the ranges of the variables used in the development of the model. Here, the range of values of annual streamflow statistics were: mean annual runoff $\mu$ [0.373, 5370] in mm; coefficient of variation of streamflow $C_q$, [0.0619, 2.97], and coefficient of skewness $\gamma$, [−2.22, 6.14]. For each of the 729 unregulated series of monthly streamflows, required storage capacities for hypothetical reservoirs were determined using the BA routing method for delivery of standardized yields in the range 0.3–0.8 in increments of 0.1 and with monthly reliabilities of 0.9, 0.95 and 0.98. A total of 12 413 estimates of reservoir storage estimates were generated and form the dataset employed in the development of our global SRY model. Iteratively reweighted least squares (IRLS) regression (see Mosteller and Tukey 1977, and Helsel and Hirsch 2002 for a description of IRLS) was then used to develop the SRY model with yield as the dependent variable. IRLS was used to minimize the impact of outliers thus producing a more robust regression model.

Using the global dataset of monthly streamflows to develop an SRY model with yield as the dependent variable led to the model:

$$Y = 0.651S^{0.203}Z_R^{-0.306}\mu^{1.135}\gamma_{-0.342}^{0.017},$$

(1)

where $Y$ is the yield, $\mu$, $\sigma$ and $\gamma$ are the mean, standard deviation and skewness coefficient of the annual inflows, all of which have units in millions of m$^3$ per year and $S$ is storage capacity with units of millions of cubic meters. Here $Z_R$ is the standardized normal variate with $R$ equal to the reliability. (For example, a system with reliability $R = 0.95$ corresponds to a value of $Z_R = 1.645$). The values of all the model coefficients are stable and their signs are consistent with our theoretical expectations. All the model coefficients are statistically significant with the smallest value of the $t$-ratio being 17.01. Thus the model coefficients in (1) are extremely precise with all p-values<0.00001. The adjusted and predicted $R$ squared are 99.2 and 99.17 respectively. This indicates both high explanatory and predictive power of the model with no observations exerting unusual influence (see Helsel and Hirsch 2002, for definition of influence). The variance inflation factors (VIFs) for the explanatory variables mean $\mu$, and standard deviation $\sigma$, were greater than ten which indicates multicollinearity between these two random variables. However given the extremely high goodness-of-fit associated with this model, combined with the large sample size used to create the model (sample size = 12 413), concerns over the high values of VIF are not warranted here (see Kroll and Song 2013). Figure 1 illustrates the goodness-of-fit of predictions of the yield model by comparing the actual yields to the predicted yields showing little or no bias. See Kuria (2014) for further cross validation analyses of this model.

There are numerous caveats associated with the generalized SRY relationships introduced here. Importantly, we ignored reservoir evaporation and seepage in the development of the SRY relationships, though in practice, estimates of such losses can be integrated into the yield requirements. In addition, seasonal variability in demand, flood control operations, and other operational factors that affect real-world reservoir yields were not considered. Finally, the relationships are based on regression so that their application should be limited to the range of values of the SRY and streamflow discharge statistics considered in their development. Those variable ranges are reported above. This is particularly important within the context of studies which explore the
impact of future climate change on water resources. Users of regression equations must always remain cognizant of danger of extrapolation of such relationships outside the bounds of the variables considered in their development.

4. Variability of reservoir water supply yield estimates

In this section, Monte Carlo experiments were performed to document the variability of water supply yield estimates derived from equation (1). Synthetic streamflows generated from a gamma (GAM) distribution were used to capture an extremely broad range of possible hydrologic conditions. The GAM distribution has broadly been recommended for approximating the probability distribution of annual streamflows in previous studies (see Markovic 1965, Vogel and Wilson 1996, McMahon et al. 2007a, 2007b and others reported therein). Reservoir inflows were characterized by their first two moments of mean, \( \mu = 1 \) and coefficient of variation, \( C_q = 0.5, 1, \) and 2. A total of 200,000 synthetic streamflows traces each of length, \( n = 10, 20, 30 \ldots 100 \) were generated. The values of \( C_q \) were selected so as to be in the range of streamflows considered in the development of the yield model as well as to capture the streamflow variability observed globally (see McMahon et al. 2007b).

To mimic what a hydrologist would do in practice, sample estimates of the mean, standard deviation and skewness of annual streamflows were then obtained from each of the 200,000 synthetic streamflow traces. The sample statistics were then used in the yield model (equation (1)) to produce 200,000 yield estimates corresponding to standardized storages in the range \( S/\mu = 1, 1.5, \ldots 5 \) (where \( \mu \) is the mean of the annual streamflows) and reliabilities \( R = 0.8 \) and 0.95. Again \( S \) and \( R \) were selected so as to be in the range of variables used in the development of the yield model.

We use the coefficient of variation of yield estimates which we term \( Cy \) to characterize the variability of water supply yield estimates, computed as the standard deviation of the yield estimates, divided by the true yield. The results of this analysis are summarized in figure 2. Interestingly, when \( Cy \) is used to characterize the variability of the yield estimates, the storage ratios (storage capacity divided by mean of the streamflows) and reliability of the yield estimates do not appear to influence the variability of the yield estimates. The length of the streamflow record and the coefficient of variation of the inflows, \( C_q \) are the only two factors that appear to influence the variability of yield estimates as described by \( Cy \). The variability of the yield estimates increases as the variability of the flows increases and decreases as the length of record increases. Apparently the variability of yield estimates gradually decreases as the length of record increases from 10 to about 40 years then it reduces gradually. These results reported in figure 2 document the impact of the increased information which results from using a longer period of record for designing reservoirs.

The results in figure 2 can be extended for use in determining the likely range of yields that can be expected for a given reservoir site on a river located in any part of the world. Determination of likely intervals associated with any random variable requires knowledge of the frequency distribution of that variable. A detailed analysis of the approximate frequency distribution of reservoir yield is presented in Kuria (2014) who documented that a three parameter GAM distribution, also known as the Pearson type III distribution (P3), provides the best overall goodness of fit to the distribution of yield estimates. However, Kuria (2014) also showed that the two parameter LN2 and GAM models also suffice for approximating the distribution of estimated water supply yield, regardless of the inflow model or record length considered since all of the values of the probability plot correlation coefficients for these models were extremely high in all cases. Considering LN2 as the distribution of the yield estimates, the \( p \)th quantile of the yield estimate is given by

\[
Y_p = \exp\left(\frac{\mu}{1} \pm \sigma_i Z_p\right),
\]

where \( Z_p \) is the standardized normal variate for a given quantile \( p, \mu = \ln\left(\frac{1}{\sqrt{1 + Cy^2}}\right) \) and \( \sigma_i = \sqrt{\ln(1 + Cy^2)} \). \( Y \) is estimated using equation (1) and \( Cy \) is obtained from figure 2 corresponding to a particular value of the coefficient of variation of the inflows to the reservoir and assumed sample size \( n \), in years.

A simple illustration is given here for the San Saba River at San Saba located in Texas USA (US Geological Survey gauging station 081460). Using annual streamflow data for 1963–2013 \( (n = 50) \), the annual statistics of the streamflow data are calculated as mean, \( \mu = 149 \), standard deviation, \( \sigma = 98.5 \), skewness, \( \gamma = 1.15 \). All units are in million cubic meters per year. Assuming hypothetical reservoirs with storage ratios \( S/\mu \) of 1, 2 and 5, and \( R = 0.9 \) and 0.95, the likely ranges of yields that can be expected for this river are shown in table 1. Here we assume that the likely range encompasses 95% of the variability in estimated yields so that \( p = 0.025 \) and 0.975 in equation (2).

5. Sensitivity of water supply reservoir yield estimates to changes in climate

Of critical interest to water resource managers is how future water supply yields will be modified under different future hydroclimatic conditions. In this section, analogously to Vogel et al. (1997, 1999b) and Brown et al. (2012) we show how the generalized SRY relationship in equation (1) can be combined with a simple hydroclimatic model to explore the impacts of changes in climatic variables on the SRY relationship.

Sankarasubramanian et al. (2001) and many others have performed generalized sensitivity analysis of water resource models using the concept of elasticity. Vogel et al. (1999a) show that each of the exponents of a power law model such as the one given in equation (1) correspond to elasticities for the corresponding independent variable, showing its non-dimensional impact on the dependent variable of interest. For example, since equation (1) is a power law model for
estimating water supply yield, the exponent 0.20 on the independent variable storage capacity $S$ can be interpreted as the storage elasticity of yield and it indicates that a 1% increase in storage capacity will increase the water supply yield by only 0.2%.

In this section we combine the yield model in (1) with a simple hydroclimatic model developed for all regions of the US. Simple hydroclimatic regression models for estimating the mean $\mu$ and standard deviation $\sigma$ of annual streamflows have been developed for the 18 water resources regions by Vogel et al (1999a). Considering USA water resource region 1, the resulting model for the mean and standard deviation of annual streamflows is given by

$$\mu = e^{-0.431 A^{0.012} \mu_p^{1.214} T^{0.5118}},$$  \hspace{0.5cm} (3)$$

$$\sigma^2 = e^{-26.39 A^{0.978} \mu_p^{1.448} T^{0.945}},$$  \hspace{0.5cm} (4)$$

where $A$ is the drainage area (km$^2$), $\mu_p$ is the mean annual precipitation (mm yr$^{-1}$) and $T$ is the mean annual temperature (Fahrenheit $\times$10). Substituting for the mean and standard deviation of the streamflows into equation (1) leads to

$$Y = e^{-35.39 S^{0.203} Z_R^{0.306} A^{0.990} \mu_p^{2.662} T^{0.433} p^{0.017}}.$$  \hspace{0.5cm} (5)$$

Since equation (5) is a power law model, its exponents may be interpreted as elasticities. For example, equation (5) implies that a 1% decrease in mean annual precipitation will result in a 2.66% decrease in water supply yields from water supply reservoirs (holding all other variables constant). We conclude that reservoir water supply yields are extremely sensitive to future changes in mean annual precipitation. Since temperature is a nonhomogeneous variable with units

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**Table 1.** The Lower Intervals (LI) and Upper Interval (UI) of reservoir water supply yield estimated from equation (1) for reliabilities $R = 0.9$ and 0.95 and storage ratios, $S/\mu = 1, 2$ and 5 San Saba River at San Saba located in Texas.

<table>
<thead>
<tr>
<th>$R = 0.9$, $S/\mu$</th>
<th>LI (m$^3$ yr$^{-1}$)</th>
<th>UI (m$^3$ yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S/\mu = 1$</td>
<td>101</td>
<td>123</td>
</tr>
<tr>
<td>2</td>
<td>116</td>
<td>141</td>
</tr>
<tr>
<td>5</td>
<td>140</td>
<td>169</td>
</tr>
<tr>
<td>$R = 0.95$, $S/\mu = 1$</td>
<td>94</td>
<td>113</td>
</tr>
<tr>
<td>2</td>
<td>108</td>
<td>131</td>
</tr>
<tr>
<td>5</td>
<td>129</td>
<td>157</td>
</tr>
</tbody>
</table>

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**Figure 2.** Coefficient of variation of yield, Cy, estimates for $S/\mu = 0.5$ and 5, $R = 0.8$ and 0.95, $C_q = 0.5, 1,$ and 2 and $n = 10–100$ for Gamma flows.
which depend upon both a constant and scale term, it is misleading to interpret the coefficient for temperature in (3), (4) or (5) as an elasticity (see Sankarasubramanian et al. 2001, section 3, for further discussion).

6. Conclusions

The goal of this study was to develop a generalized approach for estimating the yield of surface water reservoir systems and to document the uncertainty inherent in water supply yield estimates for a wide range of reservoir systems subject to the hydrologic variations and conditions which can be expected in many parts of the world. We began by developing a generalized SRY relationship for estimating water supply reservoir yield based on a global dataset of 729 rivers with a minimum of 25 years of monthly streamflows introduced previously by McMahon et al. (2007b). The global SRY regression summarized in equation (1) exhibited an extremely high level of goodness of fit, as depicted in figure 1. The reservoir yield model in (1) was further used to document the variability of the estimates of water supply yield based on actual streamflow observations. Using the coefficient of variation of the yield estimates, it was found that the storage ratios (storage capacity divided by mean of the streamflows) and reliability of the yield estimates do not influence the variability of the yield estimates. This is an extremely important result which enabled us to report the relationships in figure 2 which could be useful for describing the variability of reservoir yield estimates under extremely general conditions. Our findings indicate that the length of the streamflow record \( n \), and the coefficient of variation of the inflows, \( Cq \), are the only two factors that appear to influence Cy. The variability of the yield estimates increases as the variability of the flows increases and decreases as the length of record increases. One of the major challenges facing developing countries is to provide meaningful and stable projections of water supply yield under conditions of limited availability of streamflow data. We documented how our findings can be used to quantify the value of additional streamflow information in water supply planning and management investigations.

We also described a variety of further applications of generalized SRY relationships which have been previously reported in the literature as well as some new extensions. For example in section 5, we combined a regional hydroclimatological model with the generalized SRY relations for the purpose of evaluating the impacts of climate change on the water supply yields from water supply reservoirs. As changes in hydroclimatology continue due to changes in land use, climate and other anthropogenic influences, there will be a continued need to evaluate the impacts of such changes on water supply yields. Storage reservoirs provide one very important societal adjustment or intervention, because they enable increases in the reliability of future water supply yield. Our results provide a framework for evaluating the impacts of such future adjustments.

Acknowledgements

We are indebted to Murray Peel, Thomas McMahon and Geoffrey Pegram for sharing the global dataset of monthly streamflows upon which this study is based, and for their encouragement and support of our efforts. We are further indebted to the Fulbright Foundation and Tufts University for their support of this research.

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