Foundation identification using dynamic strain and acceleration measurements

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\textbf{A B S T R A C T}

This paper proposes a method to characterize bridge foundations using live load dynamic strain and acceleration measurements taken from substructure elements during operational loading. These measurements are used to assemble frequency response functions (FRFs) that describe the load-response behavior of the tested foundation element as a function of frequency. This method only requires that a limited number of measurements be performed on the exposed portion of the bridge substructure (piers, columns, piles, etc.), and can be performed during daily operational bridge traffic. The direct result of this analysis is an updated boundary condition that can be used for modeling of the superstructure in place of the fixed conditions typically used. Furthermore, this method can be used to identify foundation depth and type, verify design parameters, and update finite models of pile and drilled shaft behavior. A case study is presented where dynamic measurements were taken from an in-service drilled shaft.

1. Introduction

According to FHWA [9], there were 26,727 bridges in the United States over waterways or tidal areas with unknown foundations. There is also a substantial amount of bridges over land with unknown foundations, though this number is not tracked [29]. The lack of information on these bridge foundations inhibits evaluation of geo-hydraulic hazards impacting the foundation such as scour, or liquefaction. To address this potential hazard, the FHWA launched the foundation characterization program (FCP) [5,29], which also identified changes in service loads (foundation reuse), and foundation condition assessment as essential applications of foundation characterization [8]. Schaefer and Jalinoos [29] identified the following areas for foundation characterization: foundation type, pile type, embedment depth, geometry and material, integrity, and load carrying capacity. While Colin and Jalinoos [5] identify various destructive and non-destructive testing/evaluation options for identifying many of the above parameters, it was stated that “Additional research into methods of assessing the integrity and capacity of existing foundations should also be pursued.”

Maser et al. [17], Maser and Sanayei [18], and Sanayei and Maser [26] proposed that foundation stiffness be extracted from static strain, displacement, and rotation measurements from a bridge. A 2D frame model of the bridge was used to update the foundation stiffness parameter based on these measurements. While force and displacement patterns from truck crossings roughly matched expectations, parameter estimation did not identify consistent foundation stiffnesses, indicating poor estimation of the foundation boundary condition.

Santini et al. [28] extended the foundation stiffness concept to be used with dynamic data for foundation identification. This approach attempted to update a 2D FEM of a bridge subjected to dynamic loading. The foundation was modeled as a boundary condition that provided stiffness and inertial effects. The parameter estimation approach used was incapable of identifying the stiffness components of the foundation boundary condition when tested on a bridge in Texas. Several issues with the test setup and analysis approach were noted by the authors of the referenced paper.

Sipple et al. [30] used acceleration measurements from a bridge...
superstructure to identify the boundary conditions for a bridge deck. This approach did not model the substructure and updated a single boundary condition at the bearings along with structural parameters. This approach did not attempt to identify the individual behavior of the foundation, pier columns, and bearing pads. Since measurements were performed only on the bridge deck, the authors believe it will be difficult to separate foundation behavior from bearing and substructure behavior.

This paper proposes a method for measuring foundation behavior and estimating many of the parameters identified by Collin and Jalinoos [5] important to foundation characterization for known and unknown foundations. The proposed method requires only a limited number of measurements from the foundation element under consideration and does not require information regarding the superstructure or knowledge of the input excitation. Measurements are taken from a pier column or foundation wall during daily operational bridge traffic without bridge closure or special traffic considerations. The dynamic response measurements are aggregated and averaged after sufficient responses have been measured to determine the frequency response function (FRF) of the foundation. The calculated FRF is compared with an empirical model of foundation behavior to determine foundation parameters such as depth, soil properties, and foundation structural properties that govern the foundation FRF.

2. Proposed method for estimation of foundation stiffness parameters

A major novel aspect of the proposed method is that it is an output-only method that solely relies on live load excitation generated by operational bridge traffic consisting of cars, trucks, and busses. This approach allows the method to be used on in-service bridges where closure is not possible, and/or the weight of the passing vehicles are unknown. Many output-only methods (e.g., [25,30,32]) still require detailed models that predict the natural mode shapes and fundamental frequencies of the structure in order to extract information from the observed differences. This method instead creates a free body diagram (FBD) of the foundation using force and acceleration time histories at the top of the FBD calculated from strain and acceleration measurements performed on the pier column. From the operational response of the FBD to excitation, the foundation behavior can be ascertained following the method presented below.

2.1. Loading source

While operational traffic imparts primarily vertical loads onto the substructure, the dynamic response of the bridge deck generates vertical forces, lateral forces, and moments in the pier columns in a range of frequencies. As a vehicle crosses midspan, the girders underneath the vehicle deflect downward. Since the bottom girder fiber is far below the neutral axis, this deflection causes the bottom girder fibers to expand longitudinally away from the passing vehicle, inducing a lateral force onto the bridge piers. This force is secondary (and therefore expected to be smaller in magnitude) to the vertical forces directly caused by the vehicle weight. The lateral forces act on the bridge piers at a distance above the foundation, generating a moment along with any moments generated by rotation of the girders. The passing vehicles will often excite fundamental modes of the bridge deck, transferring loads onto the foundations at these frequencies. The proposed method allows this broad-spectrum forcing function to be utilized without modeling of the bridge deck and bearings with their associated fundamental frequencies and mode shapes.

2.2. Free body diagram of foundation

The proposed method examines a free body diagram (FBD) that contains a limited portion of the substructure and the foundation system. Shown in Fig. 1, the FBD consists of a lower portion of a pier column being acted upon by external forces and moments. The overlying superstructure is not explicitly considered in this analysis and is instead replaced by a time history of internal forces and moments acting on the top of the FBD. This approach significantly reduces the amount of modeling required, reducing complexity of the model, and the potential for modeling errors associated with the bridge deck, bearings, and loading source.

For a single foundation element, a total of 3 forces, 2 moments, and a torsional force will be imparted on the top of the FBD from the superstructure in a 3D reality. Fig. 1 is a 2D representation of this FBD, where out of plane forces and moments are neglected. Neglecting these DOFs is possible when there is no coupling between forces and moments acting in orthogonal directions. Torsion is neglected in this study as it is uncoupled from other DOFs, not expected to have a significant response, and due to difficulties associated with measuring torsion in a foundation column.

2.3. Instrumentation setup

Measuring the 2D operational live load-induced forces (vertical, shear) and bending moment in a foundation pier column requires a total of 4 strain gauges arranged in 2 pairs, with the top of the FBD centered between the two pairs. In 3D, to measure 5 DOF (neglecting torsion), 2 orthogonal pairs at each elevation could be used, but as few as 3 strain gauges can find both moments and the axial force. It is desirable for accurate shear force estimation to maximize the distance between the pairs of strain gauges. To avoid issues with stress concentrations, the gauges should be placed at least 1 diameter away from the top or bottom of the column pier. To measure the 3 accelerations required for 2D analysis, a setup using 4 accelerometers is proposed, with one pair of accelerometers (one vertical and one horizontal) placed on either side of the column. In reality, only a single horizontal accelerometer would be required, but the use of two accelerometers with averaging can improve the quality of the data. A schematic of the

![Fig. 1. A free body diagram of a foundation.](image-url)
proposed test setup is shown in Fig. 2.

Fig. 3 shows a typical strain and acceleration recording taken during vehicular passage. From the strain measurements, it can be observed that there is significant low-frequency (< 0.5 Hz) loading on the pier column as the vehicle passes over the instrumented pier column. Some higher frequency (5–20 Hz) excitations are visually identifiable in the raw strain measurements. The strain measurements are zeroed by setting the first sample for each recording to 0, but some drift is evident in the time history if each strain signal. This drift is believed by the authors to be due to the combination of strain gauge selection and environmental effects such as sunlight and temperature changes. Drift is frequently removed from measurement time histories by baseline correcting (detrending) the signal, but since the proposed method is frequency-based and the majority of thermal drift is at low frequencies not considered in this analysis, thermal drift is not removed from the data.

At each pair of strain gauges, the axial force and bending moment time histories due to live loading can be determined using Eqs. (1) and (2),

\[ P = \frac{AE}{2}(\varepsilon_1 + \varepsilon_2) \]  

(1)

\[ M_Y = \frac{EI}{2Y}(\varepsilon_1 - \varepsilon_2) \]

(2)

where \( A \) = transformed cross sectional area, \( E \) = modulus of elasticity of concrete, \( I \) = transformed moment of inertia, \( M_Y \) is the moment about the Y-axis, and \( \varepsilon_1 \) and \( \varepsilon_2 \) are the measured strain time histories on the left and right side of the column in Fig. 2, respectively.

Since the axial force due to live loading is constant throughout the section between the strain gauge pairs, the axial force at the top of the FBD can be calculated by averaging the axial force obtained in Eq. (1) for the top and the bottom pair, as shown in Eq. (3). While this step is in theory not necessary, it can improve the quality of the data by virtue of averaging,

\[ P_m = \frac{P_t + P_b}{2} \]

(3)

where \( P \) is the measured axial force time history, the superscript \( m \) represents the top of the FBD, the superscript \( b \) represents the bottom strain gauge pair, and the superscript \( t \) represents the top strain gauge pair. Likewise, the shear force time histories induced by live loads will be constant throughout the section, since there are no live loads applied between the two sensors. This shear force, shown below as \( V_x \) can be calculated from the moments measured at the top and bottom pair of strain gauges by dividing the difference in moments by the length (\( L \)) between the center of the two strain gauge locations, as shown in Eq. (4).

\[ V_x^m = V_x^b = \frac{M_y^m - M_y^b}{L} \]

(4)

Due to the constant shear throughout the section, the live load moment is expected to vary linearly between the two strain gauges. Since increasing the distance between the two pairs of strain gauges will increase the moment difference caused by constant shear, increasing the distance between the strain gauge pairs will provide a shear estimate that is more tolerant to sensor noise. The moment at the top of the FBD (location \( m \)) can be calculated by taking the average of the top and bottom moments (when the top of the FBD is halfway between the strain gauge pairs), as shown in Eq. (5).

\[ M_y^m = \frac{(M_y^t + M_y^b)}{2} \]

(5)

Eqs. (3)–(5) are derived from statics-based equations that neglect damping and inertial forces between the top and bottom sensor pairs (locations \( t \) and \( b \), respectively) and assume the column segment with length \( L \) is rigid. This assumption is valid as long as the section between locations \( t \) and \( b \) behaves in a stiffness-controlled manner, meaning the frequencies of interest (as discussed later in the paper) are sufficiently below the natural frequency of this section. As \( L \) is increased, the
section becomes more flexible. As the flexibility increases, the natural frequency of the system lowers, and this assumption becomes less accurate. As can be seen in the Sensors and Data Acquisition Equipment section, the distance between strain gauges used in this setup is about 1.5 times the diameter of the drilled shaft. Based on the authors’ experience, we recommend L to be between one to two times the diameter of the column.

In practice, the top of the FBD (location m) is not required to be halfway between the strain gauge pairs, as the moment can be estimated at any point in the column assuming this linear relationship and a constant shear and vertical force, but this convention makes physical sense in most cases and allows the simplification presented in Eq. (5).

To isolate the dynamic effects created by traveling vehicles or other live loading sources, the strain gauge time histories for each event are zeroed by subtracting the first sample from all the samples in the recording. It can be observed that there is some drift in the strain gauge time histories. This drift is a product of thermal sensor drift and is generally of low frequency in comparison to the frequencies of interest. Thermal drift in strain gauges is frequently accounted for by detrending or a baseline rotation of the strain time histories so that the signal begins and ends at approximately 0. Since the proposed method is performed in frequency domain and the low frequencies most affected by thermal drift are not used for updating, each strain gauge time history is zeroed, but no detrending or baseline rotation is performed.

The four measured accelerations are then converted in time domain into accelerations acting at the top of the FBD to be used in analysis. Since the top of the FBD is at the centroid of the column and the vertical acceleration measurements are performed at a distance away from the centroid, each vertical accelerometer is influenced by the vertical and rotational accelerations at the top of the FBD. The vertical acceleration is calculated as the average of the 2 vertical accelerations. The rotational acceleration is the difference between the two vertical accelerations, divided by the distance between the two sensors, as shown in Eqs. (6) and (7),

\[ A_v = \frac{A_{i1} + A_{i2}}{2} \]

\[ A_{\theta} = \tan^{-1}\left(\frac{A_{q1} - A_{q2}}{2r(d_1 + d_2)}\right) \approx \frac{A_{i1} - A_{i2}}{2r(d_1 + d_2)} \]

where \( A_i \) and \( A_q \) are the vertical accelerations measured at accelerometers 1 and 2, respectively, \( r \) is the column radius, and \( d_1 \) is the distance from the column edge to the measurement location. Rotational acceleration is defined as \( A_{\theta} \) and vertical acceleration as \( A_v \). In practice, the measured angles of rotation in radians will be small enough that the angle will be equal to its tangent, so the simplification shown in Eq. (7) can be used. The horizontal acceleration is calculated as the average of 2 accelerometer measurements, as shown in Eq. (8),

\[ A_h = \frac{A_{i3} - A_{i4}}{2} \]

where \( A_i \) and \( A_q \) are the horizontal accelerations measured at accelerometers 3 and 4, \( A_i \) is the horizontal acceleration, and the negative sign accounts for the different accelerometer orientations.

### 2.4. Response of FBD to loading

The general equation of motion (in discrete time) for the foundation element shown on the right side of Fig. 4 is given by Eq. (9),

\[ [F] = [K][Q] + [C][V] + [M][A] \]

where \( F \), \( Q \), \( V \), and \( A \) are matrices of forces, displacements, velocities, and accelerations at all degrees of freedom (DOF) in the model. These four matrices are all size 2n by m, with n being the number of DOF considered at the top of the FBD (3 for 2D, 5 for 3D when torsion is excluded), and m being the number of samples measured. \( K \), \( C \), and \( M \) are the stiffness, damping, and mass matrices, respectively and are all size 2n by 2n. In frequency domain, Eq. (9) can be rewritten in terms of measured forces, \( F \), and measured accelerations, \( A \), by expressing velocities and displacements as the first and second integrals of acceleration, as shown in Eq. (10),

\[ [F(\omega)] = \frac{1}{\omega^2}[[K][A(\omega)] + \frac{1}{\omega}[C][A(\omega)] + [M][A(\omega)]] \]

where \( F(\omega) \) and \( A(\omega) \) are matrices of Fourier transforms of the force and acceleration measurements, respectively. Both matrices are of size 2n by nfft/2, with n being 3 for 2D measurements and nfft being the length of the Fourier transform. \( \omega \) is the frequency expressed in radians and i is the imaginary operator, \( \sqrt{-1} \). \( K \), \( C \), and \( M \) are identical to the matrices in Eq. (9), and can be condensed into an FRF matrix as shown in Eq. (11),

\[ [D(\omega)] = \frac{1}{\omega^2}[[K] + \frac{1}{\omega}[C] + [M]] \]

Since measurements are only performed at the top of the FBD (half of the total FBD DOFs), it is useful to partition the matrices in Eq. (11) as shown in Eq. (12),

\[ \begin{bmatrix} [F(\omega)_{m}] \\ [F(\omega)_{u}] \end{bmatrix} = \begin{bmatrix} [D(\omega)_{mm}] & [D(\omega)_{mu}] \\ [D(\omega)_{um}] & [D(\omega)_{uu}] \end{bmatrix} \begin{bmatrix} [A(\omega)_{m}] \\ [A(\omega)_{u}] \end{bmatrix} \]

where the subscripts \( m \) and \( u \) denote the measured and unmeasured DOFs, respectively. \( F(\omega)_{m} \), \( F(\omega)_{u} \), \( A(\omega)_{m} \), and \( A(\omega)_{u} \) are all size n by nfft/2. The proposed method models the unmeasured DOFs as being supported by a set of foundation springs with no other external forces being applied at that location, so \( F(\omega)_{u} = 0 \) for all the entire n by nfft/2 matrix. The FRF matrix \( D \) is then invertible and is comprised of the stiffness components of the modeled portion of column and supporting soil springs, as shown in Eq. (13),

\[ [D(\omega)] = \begin{bmatrix} [D_{m}(\omega)_{mm}] & [D_{m}(\omega)_{mu}] \\ [D_{u}(\omega)_{um}] & [D_{u}(\omega)_{uu}] \end{bmatrix} \]

where the \( D_{m} \) entries are components of the FRF matrix for the column between the top and bottom of the right ride of Fig. 4, and \( D_{u} \) is the FRF matrix of the foundation, as discussed in the following section. Eq. (12) can be rearranged as shown in Eq. (14),

\[ \begin{bmatrix} [F(\omega)_{m}] \\ 0 \end{bmatrix} = \begin{bmatrix} [D(\omega)_{mm}] & [D(\omega)_{mu}] \\ [D(\omega)_{um}] & [D(\omega)_{uu}] \end{bmatrix} \begin{bmatrix} [A(\omega)_{m}] \\ [A(\omega)_{u}] \end{bmatrix} \]

From the bottom row of Eq. (14), the following equality shown in Eq. (9).
(15) can be obtained.

\[ A_n = -D_n \mathbf{D} \mathbf{A}_n \]  
(15)

Eq. (15) can be substituted into the top row of Eq. (14) to provide the equality in Eq. (16)

\[ F(\omega_m) = (D(\omega_m) - D(\omega_m)D(\omega_m)^{-1}D(\omega_m)A(\omega_m))A(\omega_m) \]  
(16)

For convenience, Eq. (16) can written in a condensed format, as shown in Eq. (17),

\[ F(\omega_m) = D^{FBD}(\omega_m)A(\omega_m) \]  
(17)

where \( D^{FBD} \) is an \( n \times n \) matrix whose entries in 2D are given by Eq. (18),

\[ D^{FBD}(\omega) = \begin{bmatrix} D(\omega)^{V/V} & D(\omega)^{V/H} & D(\omega)^{V/H} \\ D(\omega)^{H/V} & D(\omega)^{H/H} & D(\omega)^{H/H} \\ 0 & 0 & \end{bmatrix} \]  
(18)

where for symmetrical foundations, the entries \( D(\omega)^{V/V} \), \( D(\omega)^{V/H} \), \( D(\omega)^{H/V} \), and \( D(\omega)^{H/H} \) are generally 0, unless the substructure is angled or the foundation produces an off-centered response. In general, the proposed method is not restricted to this assumption, although off-diagonal terms that are of a different order of magnitude than the diagonal terms may be difficult to recover.

### 2.5. Foundation modeling

The bottom of the FBD is modeled as a boundary condition that represents the foundation behavior. The boundary condition isn’t assumed to impose external loading on the FBD, only react to loading applied at other DOFs. The boundary condition consists of a frequency-dependent stiffness and damping commonly modeled as a spring and dashpot. This representation is a commonly used approach for modeling many types of foundations, including foundations with complicated subsurface geometry, as shown in Fig. 4.


In general, these methodologies describe the relationship between force applied to the foundation and displacement of the top of the foundation. Since they are developed using simplified models of foundation behavior, it is assumed that no coupling exists between the vertical and rotational DOFs. The stiffness and damping components obtained using one of the above-mentioned methodologies can be arranged into a frequency-response function (FRF) matrix that represents the foundation, \( D_F \), as shown in Eq. (19).

\[
D_F(\omega) = \begin{bmatrix} -\frac{K_F(\omega)}{\omega^2} + \frac{C_F(\omega)}{\omega} & 0 & 0 \\ 0 & -\frac{K_F(\omega)}{\omega^2} + \frac{C_F(\omega)}{\omega} & 0 \\ 0 & 0 & -\frac{K_F(\omega)}{\omega^2} + \frac{C_F(\omega)}{\omega} \end{bmatrix}
\]  
(19)

The matrix shown in Eq. (19) is the foundation FRF and is used in Eq. (13) as the boundary condition at the bottom (location \( u \)) of the free body diagram. Once identified, this boundary condition can be placed into a dynamic model of the entire bridge to simulate the appropriate boundary condition and improve the accuracy of that model. The division by \(-\omega^2 + i\omega\) in the stiffness and damping terms (respectively) follows from Eq. (11), where integration is performed to relate forces and accelerations.

Since the validation model used for this research is a drilled shaft with a slenderness ratio of about 11, it is most appropriate to model the foundation as a pile [14], following the approach discussed by Novak [22] and Novak and Aboul-Ella [23]. The important parameters used in this approach include the length and slenderness ratio of the pile, the soil stiffness of the surrounding soil, and the stiffness of the pile. This approach is believed to be best suited to identifying the length of the pile, as length has a varying non-linear relationship with deformations at the vertical, horizontal, and rotational degrees of freedom. Changing any of the input parameters used for the previously described models (e.g. pile stiffness, pile length, soil shear modulus for the model used in this paper) alters the shapes of the FRFs when plotted as a function of frequency. For the models used in this validation study, an increase to the soil shear modulus causes a general increase to the foundation FRF values, an increase in pile length leads to a general increase in the ratio between vertical and horizontal foundation FRFs, and an increase to pile stiffness causes a reduction to the ratio of the vertical FRF divided by the horizontal FRF. Since these parameters impact the various FRF values in different ways, they are not colinear with respect to the objective function can all be estimated from a set of measured data.

### 3. Automated event selection

A major novel aspect of the proposed method is that it can be paired with a long-term instrumentation setup to provide a continual snapshot of foundation behavior. As part of this research selection criteria have been devised so that foundation behavior can be automatically extracted from ambient data acquired from a long-term instrumentation setup. The automated procedure allows determination of foundation behavior from a limited set of data (in the range of several hours to days) automatically with no human intervention. Such a system could potentially identify changes to the foundation, such as scour, deterioration, or seismic damage. The procedure for automatic data collection is as follows:

- Recordings are performed for a long segment of time (e.g., several hours or days)
- Events (passage of one or several vehicles) are separated from the extended recording
- A time history of forces and accelerations is calculated for the selected windows
- The magnitude-squared coherence (MSC) between forces and accelerations is used to identify frequencies in each event with good signal to noise (SNR)
- Short-time Fourier transforms (STFT) are made of each event to identify short time windows (~4 s) where the force and acceleration measurements exceed background noise by a defined amount
- The foundation FRF is extracted from data from the selected windows

The MSC and STFT selection criteria are necessary for use in the proposed method, as the recordings observed at an in-service bridge indicate that much of the data observed is at low SNRs, necessitating a large amount of processing to identify portions of recordings with the best SNR. The level of signal processing used in this paper to extract only the most heavily excited frequencies is not typically employed on monitoring of civil structures but allows for extraction of meaningful data from signals that might otherwise provide highly variable results.
3.1. Event separation

Periods of increased dynamic loading on the foundation from the extended recording, referred to herein as events, are separated from the extended recording for further analysis. Event selection is performed on the basis of variance in the acceleration signal measurements, as the signal variance is a simple calculation that provides a measure of the total power in a signal [31]. Truck crossings, vehicular traffic, and other operational excitations will impart dynamic loading on the foundation resulting in higher recorded power in the accelerometer signals. The event separator reduces hours of continuously sampled data into a limited number of high energy events where further analysis is performed.

The variance of all accelerometer time histories is calculated in 1-second increments and summed together for all recorded time intervals. Two thresholds are set: a peak threshold that must be crossed for an event to be identified, and a secondary threshold that specifies the data surrounding the peak to be retained. An additional 3 s of samples on either side of every event is retained during this step. It is desirable to use easily exceeded thresholds at this stage as the goal of the first selector is to select a reasonable amount of data for further analysis.

3.2. Event frequency selector

Once events have been separated, time histories of the forces, moments, and accelerations at the top of the FBD are calculated, as described in the proposed method section. The second selector is applied to these calculated time histories to distinguish which frequencies contain usable data for each separated event. The basis of selection for this selector is the magnitude-squared coherence (MSC) between the calculated force and vertical acceleration time histories, as well as the multiple-input multiple-output MSC between the horizontal force, moment, horizontal acceleration, and rotational time histories. For a single-input single-output system (as is the assumed case for vertical motion), the MSC is calculated by dividing the cross-power spectral density of the two signals by the product of each signal’s power spectral density, as described by digital signal processing books, such as Alessio [1]. The result is a function of frequency with each frequency bin containing a value ranging from 0 to 1. If the two signals are perfectly correlated and noise free, the MSC will be 1 for all frequencies. Uncorrelated signals or situations where the output is a function of multiple inputs will produce a lower MSC, though it may not be 0. In the presence of noise, the MSC will tend to be reduced by the random nature of signal, therefore use of a threshold MSC when selecting data helps ensure that a high SNR is maintained [1]. This concept can be extended to multiple-input, multiple-output systems where the MSC relates each output to all considered inputs.

3.3. Window selector

Once an excitation event has been identified and frequencies of interest have been chosen for that event, localized windows are selected within each event where the calculated force and accelerations at the top of the FBD exceed the background noise in a specific frequency bin by a specified amount. The selection of windows is performed using estimated values of the soil and foundation parameters. Eqs. (15)–(17) show the equation proposed to provide a weighted average of the FRF for a specific frequency bin,

\[
FRF(\omega)_{VV} = \frac{\sum_{i=1}^{nWin(\omega)_{VV}} 20 \log |F_i(\omega)| |A_i(\omega)|}{\sum_{i=1}^{nWin(\omega)_{VV}} |F_i(\omega)| |A_i(\omega)|}, 
\]

where \(F_i(\omega)\) and \(A_i(\omega)\) are the FFTs of the \(i^{th}\) window of measured vertical forces and accelerations at frequency \(\omega\), respectively. \(nWin(\omega)_{VV}\) is the number of vertical measurement windows selected at frequency \(\omega\), and \(FRF(\omega)_{VV}\) is the average measured vertical frequency response function for the FBD. Since the remaining 4 non-zero DOFs are coupled, they must be solved for simultaneously. Therefore, it is necessary to provide an initial estimate of the \(D_{FBD}\) shown in Eqs. (15)–(17). The initial estimate of this matrix is calculated using the equations outlined in Novak [22] using estimated values of the soil and foundation parameters. Eqs. (21)–(24) provide the proposed equations for calculating the weighted frequency bin average for the remaining non-zero FRF terms.

\[
FRF(\omega)_{HH} = \frac{\sum_{i=1}^{nWin(\omega)_{HH}} 20 \log |F_i(\omega)| |A_i(\omega)|}{\sum_{i=1}^{nWin(\omega)_{HH}} |F_i(\omega)| |A_i(\omega)|}, 
\]

\[
FRF(\omega)_{HZ} = \frac{\sum_{i=1}^{nWin(\omega)_{HZ}} 20 \log |F_i(\omega)| |A_i(\omega)|}{\sum_{i=1}^{nWin(\omega)_{HZ}} |F_i(\omega)| |A_i(\omega)|}, 
\]

\[
FRF(\omega)_{ZH} = \frac{\sum_{i=1}^{nWin(\omega)_{ZH}} 20 \log |F_i(\omega)| |A_i(\omega)|}{\sum_{i=1}^{nWin(\omega)_{ZH}} |F_i(\omega)| |A_i(\omega)|}, 
\]

\[
FRF(\omega)_{ZV} = \frac{\sum_{i=1}^{nWin(\omega)_{ZV}} 20 \log |F_i(\omega)| |A_i(\omega)|}{\sum_{i=1}^{nWin(\omega)_{ZV}} |F_i(\omega)| |A_i(\omega)|}.
\]
are the horizontal and rotational excitations, respectively. The average frequency response function for the horizontal, rotational, and 2 cross-rotational DOFs, respectively, are the number of horizontal/rotational windows identified for each frequency, equal because these DOFs are solved simultaneously. \( F_\text{H} \) and \( F_\text{D} \) are the FFT of the horizontal forces and moments for the \( i \) th window of frequency \( \omega \), respectively. \( A_\text{H} \) and \( A_\text{D} \) are the FFT of the horizontal and rotational accelerations for the \( i \) th window of frequency \( \omega \), respectively. The \( D_{\text{FRD}} \) terms in Eq. (21)–(24) are components of the matrix in Eq. (18). Since this matrix is symmetrical, an average of the 2 cross terms, as shown in Eq. (25) is used.

\[
FRF(\omega)_{\text{H}} = \frac{\sum_{i=1}^{n_{\text{Win}}(\omega)_{\text{H}}} 20 \times \log_{10} \left[ \frac{|F_\text{H}(i)| - D_{\text{H}}(i)}{D_{\text{D}}(i)} \right] \times |A_\text{H}(i)|}{n_{\text{Win}}(\omega)_{\text{H}}} \tag{23}
\]

\[
FRF(\omega)_{\text{D}} = \frac{\sum_{i=1}^{n_{\text{Win}}(\omega)_{\text{D}}} 20 \times \log_{10} \left[ \frac{|F_\text{D}(i)| - D_{\text{D}}(i)}{D_{\text{D}}(i)} \right] \times |A_\text{D}(i)|}{n_{\text{Win}}(\omega)_{\text{D}}} \tag{24}
\]

where \( FRF(\omega)_{\text{H}}, FRF(\omega)_{\text{D}}, FRF(\omega)_{\text{H},\text{D}}, \) and \( FRF(\omega)_{\text{D},\text{H}} \) are the average frequency response function for the horizontal, rotational, and 2 cross-rotational DOFs, respectively. \( n_{\text{Win}}(\omega)_{\text{H},\text{D}} \) are the number of horizontal/rotational windows identified for each frequency, equal because these DOFs are solved simultaneously. \( F_\text{H} \) and \( F_\text{D} \) are the FFT of the horizontal forces and moments for the \( i \) th window of frequency \( \omega \), respectively. \( A_\text{H} \) and \( A_\text{D} \) are the FFT of the horizontal and rotational accelerations for the \( i \) th window of frequency \( \omega \), respectively. The \( D_{\text{FRD}} \) terms in Eq. (21)–(24) are components of the matrix in Eq. (18). Since this matrix is symmetrical, an average of the 2 cross terms, as shown in Eq. (25) is used.

\[
FRF(\omega)_c = \frac{FRF(\omega)_{\text{H}} + FRF(\omega)_{\text{D}}}{2} \tag{25}
\]

After the FRF frequency bin averages for each DOF are obtained, outliers are removed from the data to limit their impact on the analysis. Very few outliers were identified for vertical FRF estimation, and most horizontal/rotational outliers observed were in cases where the horizontal and rotational excitations were of very low magnitude. Presently, outliers are considered to be all points where the FRF would be greater than 10 dB away from the calculated mean, with the horizontal data being rejected if any measurement is outside the range of ± 10 dB of the mean. After this process is performed, the average is calculated again without the outlier data present. A more sophisticated method of outlier data rejection is to remove measured data points outside the range of ± 2 standard deviations away from the mean, although this is not possible for all frequency points as some did not have enough data collected to determine a standard deviation. Approximately 4% of the total windows obtained in the validation case were discarded as outliers, in line with the 95% interval covered by ± 2 standard deviations.

The differences between the average FRF calculated from data and the \( D_{\text{FRD}} \) matrix calculated as shown in Eq. (19) are then minimized. To perform this minimization, an objective function that minimizes the difference between the estimated FRFs and the assumed FRF is proposed in Eq. (26),

\[
J(G, L, E_p) = \sum_{D_{\text{DOF}}} \sum_{\omega=1}^{N_{\text{y}}} \frac{n_{\text{Win}}(\omega)_{\text{DOF}} \times (|FRF(\omega)_{\text{DOF}} - D_{\text{FRD}}(\omega)|^2)}{\text{sum}(n_{\text{Win}}(\omega)_{\text{DOF}})} \tag{26}
\]

where \( N_{\text{y}} \) is the frequency bin corresponding to the Nyquist frequency (half of the sampling rate), and there are half as many frequency bins as sample length. Only frequency bins where data was selected using the preceding procedure are considered in the objective function. Eq. (26) weighs each frequency bin by the number of observations in that bin, so that bins with a greater number of observations have a higher weight factor than bins with a lower number. For this research, the fmincon routine in MATLAB [20] was employed to find the \( D_{\text{FRD}} \) matrix that minimizes Eq. (26). Typical or expected values of input parameters are used to develop the analytical \( D_{\text{FRD}} \) curves found from literature. Lower and upper bounds were supplied to the routine to ensure convergence at the global minimum. It was observed that very wide bounds following reasonable assumptions about foundation parameters were suitable.

4. Validation of proposed method

To validate the proposed method, an instrumentation scheme was implemented on a test bridge with a known foundation. Measurements of the dynamic response of the foundation were performed during operational traffic and parameter selection was performed with the resulting data obtained.

4.1. Powder Mill Bridge

The Powder Mill Bridge (PMB), shown in Fig. 5, is a 3-span continuous steel girders bridge located in Barre, MA. The PMB has 6 girders and is approximately 47 m long and 11.8 m wide with two 5 m wide travel lanes and a 1.8 m wide sidewalk. The northern span (on the left side of the figure) contains two additional girders to accommodate a wider deck with turning lanes. The two interior pier bents each consist of a 1.2 m wide by 1.0 m deep concrete pier cap supported by 3 920 mm diameter columns, each supported by 1.07 m diameter drilled shafts. The drilled shafts are surrounded by a medium to stiff overconsolidated clay and are terminated in bedrock approximately 11.5 m below ground surface. The southern interior bent (shown during construction in Fig. 6) is located near an electrical junction installed alongside long-term monitoring equipment during the bridge’s construction [27]. The bridge experiences moderately frequent truck traffic, in part due to a nearby landfill. The PMB was chosen as a test case due to its accessibility, the long pier
columns that allow for easy instrumentation, and the commonality of the bridge type.

### 4.2. Sensors and data acquisition equipment

The equipment used for the PMB validation test consisted of 4 BDI ST350 strain transducers [3], 4 Wilcoxon 731A/P31 seismic accelerometers and amplifiers [33], and an NI BNC 2110 data acquisition system (DAQ). The strain transducers were installed on the left column of the southern pier in Fig. 6 with accompanying extension bars [3] to allow strains to be measured over a distance of 0.457 m (18″). Since the ST350 functions as a load cell with an output controlled by the amount of force applied to the transducer, the use of an extension bar provides an amplified signal that can be correlated to the average strain over the measurement distance. As the shear due to live load is expected to be constant over the measurement distance, a linear strain profile will be expected and the output of ST350 is taken to be the strain at the midpoint of the sensor and extension bar. This style of strain gauge transducer allows for extremely precise strain measurements but is prone to thermal drift as changes in the temperature of the extension bar will cause elongation and shortening that impacts the measurements. These same sensors can be installed into anchor bolts connected to the concrete to provide a permanent installation.

The strain transducers were connected to a Micro-Measurements 2120B dynamic strain gauge signal amplifier and conditioner. Fig. 7 shows a view of two sides of the instrumented pier column during testing. Fig. 8 shows a school bus crossing the PMB during testing, with the instrumentation setup visible on the column beneath the bus in the lower left corner.

### 4.3. Event separation

Approximately 8 h of recordings were performed at the PMB with the instrumentation setup proposed in this research. The event extraction routine was applied to the recorded data to separate periods of increased excitation. The two variance thresholds used to extract events were $2 \times 10^{-6}$ Volts$^2$/sample for the peak threshold and $2 \times 10^{-7}$ Volts$^2$/sample for the lower threshold. Three seconds on either end of the lower threshold were extracted along with the portions that exceeded the variance threshold. The appropriate threshold will be specific to the bridge being investigated, the background noise level, and the expected level of excitation. Fig. 9 shows an accelerometer signal alongside the calculated average variance of accelerometer signals for a typical traffic event.

Fig. 10 shows a time history of forces acting on the top of the FBD, calculated from strain measurements taken during this event using Eqs. (1)–(5). It can be observed that the vertical force loading on the foundation is of significantly greater magnitude than the horizontal force loading. The noise in the horizontal force and moment time histories appear to be greater in comparison to the signal than with the vertical force time history. This is expected, as the horizontal and moment loading is largely secondary effects for the vertical loading, and the horizontal time history is calculated without any of the spatial
averaging performed for the moment and vertical force time histories.

As can be seen in Fig. 10, the drift observed in the strain gauge time histories (Fig. 3a) is carried into the force and moment time histories. As mentioned earlier, low frequency drift is commonly corrected using linear detrending or high-pass filtering which is not needed since the low frequencies where interference is observed can simply be neglected. Fig. 11 shows the accelerations at the top of the FBD calculated using Eqs. (6)–(8) for a selected event.

4.4. Frequency identification

After separation of events and calculation of the force and acceleration time histories, the MSC selector is used to identify the frequencies for each event that meet a minimum level of coherence (0.90 for this research). Only the frequencies which exhibit high coherence over the length of the event are considered. If no frequencies exceeded this threshold, no data from the event selected is utilized. Due to the random nature of the induced excitation, the relatively low signal levels, and the lack of forcing at frequencies not excited by the bridge deck, many frequencies from each recording are not selected during this step. As can be observed from Fig. 12, for a selected event, only a small portion of frequencies in the lower range meet the coherence threshold. Of the 303 events isolated during the previous step, 255 of them had at least one frequency meet the MSC threshold criteria.

4.5. Window selection

For this research, two thresholds were chosen as criteria for window selection: 6 dB for horizontal/rotational forces/moments/accelerations,
and 12 dB for vertical forces/accelerations. The vertical and horizontal windows were considered separately so that windows that exceeded the vertical criterion were retained for vertical analysis, and windows that exceeded the horizontal criterion were retained for horizontal/rotational analysis. Only windows where both the force and acceleration measurements exceed their background by the threshold were considered. The lower threshold for horizontal rotational DOFs was selected due to the lower amount of excitation caused by a passing truck in these directions. The background noise is determined by taking a periodogram of the measurements when no traffic or other discernable and significant external excitations were overserved. Forces and accelerations were calculated from measurements obtained during a 70-second long excitation-free period at 256 samples per second. The thresholds considered for this example are very low (corresponding to ~2× and 4× the noise level). In practice, a higher threshold would be desired, but this was not possible due to the low SNR observed in the strain gauges. Newer strain gauges than used for this example are commercially available with lower electronic noise and higher sensitivity. Also, many bridge types with slenderer columns and heavier traffic loads will experience higher strains in the strain gauges.

A visualization of the STFTs for a typical event is given in Fig. 13 for vertical acceleration and Fig. 14 vertical force measurements. The same process is applied to horizontal and rotational DOFs, with only measurements where both exceed the criteria considered.

For the event shown above, a total of 17 frequency bins (out of 257) were considered on the basis of the MSC. Of the 255 events with at least some frequencies meeting the MSC threshold criterion, 202 events had at least 1 window meet the magnitude selection criterion. From these 202 events, a total of 1600 vertical excitation windows were extracted, and 437 horizontal/rotational excitation windows were extracted. It should be noted that a new window is created for each time and frequency, so some points in time have several windows stored correlating to different frequencies.

4.6. Averaging frequency bins

As described in the automated event selection section, outliers were automatically removed after initial determination of the average for each frequency. A total of 17 horizontal windows out of 436 were discarded. Fig. 15 shows a plot of all 419 horizontal FRF windows that were selected for analysis (circles), along with the averages for each frequency bin where windows were identified (stars).

4.7. Estimation of foundation stiffness parameters

Three foundation stiffness parameters were considered during updating for the validation model: the depth of the foundation below grade, the shear modulus of the soil, and the modulus of elasticity of the material. The material properties were estimated using a combination of the frequency-domain data and the time-domain data. The frequency-domain data were used to determine the natural frequencies and mode shapes of the bridge, while the time-domain data were used to determine the damping ratios of the bridge. The foundation stiffness parameters were then adjusted until the predicted response matched the measured response as closely as possible.

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pile. These three parameters were chosen as their selection will greatly impact the behavior of the foundation, they are expected to be important parameters to estimate for unknown foundations, and they are known with a relative degree of confident for the validation case. Table 1 provides the parameters which provide the optimal curve fit for the validation model.

The initial value chosen for the pile length and stiffness are based on construction design plans with bedrock elevations (from nearby borings), and compression test breaks of the concrete cylinders used to form the pier columns. The soil shear modulus for the hard clay present at the bridge typically ranges from 8 to 20 MPa. Convergence to the global minimum, was examined by starting from various initial values. It was found that parameter identification converged to identical values when the pile length, soil shear modulus, or pile stiffness was varied by an order of magnitude. The updated parameters returned from the parameter estimation routine at global convergence also were in very good agreement with the initially expected values, as shown in Table 1. For the PMB validation case, the proposed method appears to have high predictive capability. Plots of the obtained FRFs versus the best fit curves are provided in Fig. 16.

As can be seen, the fitting routine finds a set of physical parameters that well matches the expected behavior of the foundation to the observed behavior. Some divergence between the measured FRF and the analytical FRF can be observed in Fig. 16(a), however those are in frequency bins where few observations (as low as 1) were observed. Due to the low number of observations, these bins are expected to have higher variance and are weighted lower following Eq. (26).

A major assumption that can influence the calculated internal forces using the measured strains is the modulus of elasticity of concrete. We have used the 28-day concrete cylinder tests to calculate the modulus of elasticity. A study was performed to evaluate the sensitivity of the results to inaccurate parameter assumptions, such as the modulus of elasticity of the concrete. Since the concrete modulus is an assumed parameter used to calculate the force acting upon the foundation, an inaccurate assumption of this parameter will lead to an inaccurate forcing function time history. It was found that varying the concrete modulus of elasticity (and the subsequent change to the transformed properties used in the analysis) most heavily impacted the estimation of the soil shear modulus and pile stiffness. The pile length estimation was not largely impacted by varying the concrete modulus of elasticity, believed to be because the pile length has varying non-linear correlations to various elements of the foundation stiffness matrix assembled following Novak [22]. As a result, it is recommended using accurate measured modulus of elasticity of concrete based on available NDT methods, however, this does not change the validity of the proposed methodology based on using strain measurements and the free-body concept.

### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expected Value</th>
<th>Converged Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile Length (Lp)</td>
<td>11.5 m</td>
<td>12.0 m</td>
</tr>
<tr>
<td>Pile Stiffness (Ep)</td>
<td>25.7 GPa</td>
<td>20.6 MPa</td>
</tr>
<tr>
<td>Soil Shear Modulus (G)</td>
<td>8–20 MPa</td>
<td>11.1 MPa</td>
</tr>
</tbody>
</table>

Fig. 16. Measured (a) vertical, (b) horizontal, (c), rotational, and (d) cross FRFs versus best fit analytical FRF.

5. Conclusions

A method has been presented that allows for foundation behavior to be accurately modeled following anticipated empirical formulae. The method was employed in a validation example to verify that the proposed measurements could be performed, that the automated setup would be capable of extracting foundation information, that the calculated foundation behavior matched observed behavior, and to validate the predictive capabilities of the model. It was determined through testing that the required measurements were able to be performed with commercially available and practical sensors, and that the required excitation level could be achieved through arbitrary excitation related to normal, operational vehicle traffic.

The automated method was capable of parsing an entire day of data
and extracting useful information with minimal computational overhead. While the initial programming of the methodology requires a high degree of signal processing knowledge, the method can be programmed in such a way that data is automatically parsed at regular intervals, analysis automatically performed, and engineers provided with regular updates of the calculated FRFs and estimated foundation parameters. Once programmed initially, this setup can be applied to various bridges by changing the appropriate parameters for providing frequency response functions that correspond to that foundation.

The proposed method estimated a pile length nearly identical to the expected pile length, a soil shear modulus within expectations, and a pile modulus of elasticity approximately 80% of the expectation. The soil modulus is normally highly variable. Overall the predictions reasonably validated the proposed method.

6. Future work

The automated collection approach allows this method to be adapted for programs designed to provide continuous or intermittent monitoring of foundation performance. Some important issues that can potentially be identified include scour monitoring and monitoring of pile deterioration. Preliminary work shows that a foundation undergoing scour or loss of cross-sectional area will produce a detectable change in the foundation frequency response function. For a long-term monitoring system, the sensors would need to be placed in an area relatively protected from the elements and above potential water elevations during a scour event. Future work to enable this usage would require identifying the statistical sensitivity of the method to changes in important parameters such as pile length, soil stiffness, pile modulus, and elevation of soil surrounding the pile. The work in the paper can be extended beyond drilled shafts and single piles and be applied to various types of foundation systems with other dynamic stiffness formulations.

Declaration of Competing Interest

The authors declared that there is no conflict of interest.

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.engstruct.2019.109811.

References


Engineering Structures xxx (xxxx) xxxx


Appendix A. Supplementary material

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Further reading