Efficient impedance model for the estimation of train-induced vibrations in over-track buildings

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Abstract
Rapid growth and the urban development of Chinese megacity in recent decades have led to land utilization challenges and the need for over-track buildings above metro depots that are part of the subway systems in these cities. Frequent subway operations into and out of the depots, which generate excessive vibration that transmits into the over-track buildings, can have adverse effects on the buildings’ occupants in terms of feelable vibration and noise. An efficient modeling procedure is presented for estimating vibration levels within the buildings before construction to determine whether vibration mitigation is required. Models were developed and validated based on vibration measurements in low-rise and high-rise buildings over a metro depot in Shenzhen, China. Impedance models were developed that characterize vibration transmission through the primary support columns or load-bearing walls into the building floors. Inputs to the model are vibration levels at the buildings’ foundation where the model estimates the transmission through individual support structures to the upper floors. Vibration estimates at different elevations within the building were compared with the measured levels during train pass-bys under the buildings with good agreement. Based on axial wave transmission impedances for column or load-bearing wall segments and simple impedance expressions for building floors, the closed-form analytical impedance model is an efficient and cost-effective tool for estimating floor vibration before construction.

Keywords
Impedance modeling, axial wave transmission, train-induced vibration, human comfort, concrete building with load-bearing wall, steel-framed building, high-rise and low-rise buildings

1. Introduction
Urban rail transit is an efficient way to reduce highway traffic and protect environment in metropolitan cities. As of the end of 2018, 35 cities in mainland China have in-service rail transit lines with a total of 5761 km. Compared with 2015, an extra 2143 km of urban rail transit had been put into operation. In addition, 53 Chinese cities are expected to build additional 6374 km of urban rail transit lines in the next few years. Figure 1 illustrates the rapid development of urban rail transit in China (The China Urban Rail Transit Association, 2019).

With the large scale of construction in the next few years, the government investment will be significant with a long payback period. However, operating losses in the urban rail transit system are common in almost cities of China except Hong Kong. Construction financing is a core issue that urgently needs to be solved. In addition, as the cities continue to expand, utilization efficiency of urban land and space resources becomes more important. According to the current design standards of the urban rail transit of China (Zou et al., 2015), urban rail transit lines should be equipped with metro depots as a logistical support center for day-to-day operations. The metro depot is a subway facility where trains regularly park for maintenance, testing, and storage,
and usually occupy 250,000–450,000 m² (Ministry of Housing and Urban-Rural Development of the People’s Republic of China, 2013). Traditional metro depots, without over-track buildings and which require large open space, can significantly limit the available space in the depot. Lower density buildings are generally not conducive to an economical use of urban land.

To provide funds for subway projects and improve utilization efficiency of urban land and space resources, more than 27 Chinese ultra-class and first-class cities are designing and building superstructures over new metro depots. The new metro depots (Figure 2) incorporate both the tracks, stations, and over-track building complexes which include shopping malls, hotels, office buildings, apartments, and other public utilities. These modern transit-oriented developments comprehensively use land resources in urban spaces and result in smaller building footprints (Sun et al., 2017).

According to the spatial relationship between the metro depot and the ground, three different types of metro depot superstructures are defined as underground type, ground type, and elevated type, as shown in Figure 2. In the underground type, the depot and tracks are below ground. Elevated metro depots are located within the building superstructure and use the spaces below and above for development (Guo et al., 2017). The advantage is that the superstructures are well integrated and in harmony with the surrounding urban environment. However, few depots are of the underground or elevated types because of the high costs of construction. For ground type depots, the tracks at ground level are covered by large structural platforms typically 9–16 m above ground that are supported by load-bearing columns/walls set in between train tracks. The platform level is for car parking and supports the buildings above. The ground type is the most common type, has the advantage that the construction is less difficult, and the investment is relatively small.

Countries such as Britain, the United States, Japan, and Singapore are further along in developing building structures over urban rail transit systems (Wallace and Ng, 2016; Zhou and Zhao, 2016). However, the development of the metro depot superstructures in China is at an earlier stage. Some individual projects have experienced problems with vibration and noise. The large number of subway operations into and out of the depots can generate excessive vibration levels that transmit into the over-track buildings and can adversely impact the buildings’ occupants in the form of feelable vibration and noise as well as hamper the performance of vibration-sensitive equipment.

How to effectively reduce and eliminate the environmental problems caused by vibration has become a key factor that limits the successful development of metro depot superstructures in China. Most vibration prediction methods are based on empirical models and past design experience. Empirical modeling accounts for the distance between the building and the tracks, wheels, and rail dynamic forces, train speeds, vibration attenuation and other factors (Connolly et al., 2015; Federal Railroad Administration, 2012; Verbraken et al., 2011). For the new metro depots before construction, vibration source levels can be estimated from past measurements at existing depots, and potential vibration impacts can be assessed based on Chinese guidelines (Ministry of Environmental Protection of the People’s Republic of China, 2008). Detailed predictions
are needed before construction begins during the initial scoping assessment which can identify vibration-sensitive areas and area appropriate criteria.

Connolly et al. (2015) reported that the majority of vibration assessments were performed based on measured transfer functions describing vibration transmission into the buildings due to train operations. There is a need for a greater number of field measurement studies of vibration issues in metro depots and buildings. Measurements of vibration transmission from the track through the soil into building structures are limited. Most studies have used numerical simulation to predict building vibration.

Numerical simulation needs to integrate three factors: the vibration source, the vibration transmission path, and the building structure. The analysis of vibration sources has matured, involves the calculation of dynamic loads using a train–track model which accounts the train weight, track irregularities, impulse irregularities caused by rail and wheel wear, and continuous irregularities due to eccentric wheel mounting, etc. (Kouroussis et al., 2014). The dynamics of the train car is often described by 4-degree-of-freedom (DOF), 9-DOF, or even 78-DOF models that include the body, bogie, and wheelset (Costa et al., 2012; Kouroussis et al., 2012; Wallace and Ng, 2016; Zhai et al., 2009). The track structure is usually described by two or three dimensional models, including one, two, or multilayer structures, respectively, to simulate rail, rail pad, fastener, sleeper, ballast, foundation, and other materials (Cao et al., 2011; Ju et al., 2010). The overall train–track model is configured to analyze the effects of wheel–rail irregularities, turnouts, and rail joints (Lei and Rose, 2008).

For ground type metro depots, the train-induced vibration transmits from the track structure through the soil into a buildings’ foundation in the load-bearing columns/walls. Geophysical measurements to determine soil properties are needed to determine whether the soil models are correct or not (Connolly et al., 2015). Geotechnical issues are complicated; it is better to perform the in situ geophysical tests to determine the soil properties. Lopes et al. (2014) stated that the vertical resonance frequency of the building is related to the interaction between the soil and the structure that depends on the shear wave velocity of the soil. The higher the shear wave velocity, the larger the resonance of the building-ground system. Another challenge is the quantification and acquisition of soil–structure coupling parameters and building coupling loss (Kuo et al., 2019).

Kuo et al. (2019) confirmed by the research on the building vibration along the railway that when the vibration caused by the train propagates into the building through the building foundation, the vibration level may be amplified or reduced, that is the frequency-dependent coupling between soil and building structure needs to be taken into account. Vibration levels within a building depend on the nature of the foundation, weight, and the soil type (Xia et al., 2005). Different from the common train-induced building vibration and noise problems (Kouroussis et al., 2011; Yang et al., 2018), the subway trains in a metro depot operate directly underneath over-track buildings, with the result being potentially greater transmission of vibration into and noise within the buildings. However, reports predicting vibration transmission into over-track buildings in metro depots are few in number. Compared with the experimental methods, finite element models can require significant time to develop as well as to run the computations (Gao et al., 2015; Xu et al., 2015). In addition, because of the uncertainty of the numerical model parameters, additional runs may be required to establish the sensitivity of the predictions to variations in different model parameters. To reduce computing time, a lot of studies have been carried out to simplify and optimize the modeling for finite and boundary element models (Galvin and Dominguez, 2009; Sheng et al., 2006; Verbraken et al., 2011), 2.5 dimensions
finite element models (François et al., 2010; Liang et al., 2017), finite difference models (Katou et al., 2008), dynamic stiffness matrix methods (Guo et al., 2017), semi-analytical models (Sheng et al., 2004) and theoretical models (Cao et al., 2017, 2018).

Train operations on test lines and in throat areas were the main sources of vibration for studies of metro depots with over-track buildings that included measurements as well as estimates based on simplified analytical building models (Zou et al., 2015, 2017). Simplified one-dimensional impedance models were developed to predict the propagation of axial, that is longitudinal, vibration within the buildings’ support structures before construction during the design phase (Sanaye et al., 2012, 2014; Zou et al., 2018). The models estimate building floor vibration because of measured inputs at the foundation.

Another approach is to model the building, including all details (e.g. openings, staircases, elevator shafts, columns, beams, slabs, and walls), using finite element analysis. The cost and effort to develop such models can be substantial where building owners and designers are often reluctant to commit to such a formidable task early in the design process. Impedance models can more effectively be used than finite element modeling and have been validated for a 4-story building with columns as well as for 14-story and 25-story frame-shear wall structure buildings (Zou et al., 2018).

This study extends earlier impedance models by the authors to account for the transmission through the columns and load-bearing walls commonly found in the over-track buildings. Two full-scale field building tests during train pass-by events were performed to compare estimates using impedance models with the measured vibration levels. The dominant mode of transmission of train-induced vibrations into the buildings are due to axial waves that propagate vertically within the columns and load-bearing walls, and bending waves in columns and load-bearing walls are sufficiently reflected by the impedance of the floors relative to bending wave impedances in the columns and load-bearing walls.

2. Mathematical impedance model

This section presents a one-dimensional impedance model of axial wave vibration transmission in the support structures, that is columns or load-bearing walls, in buildings (Zou et al., 2018). Vibrational energy radiates out into the floors in the form of transverse bending waves due to the vertical displacements at the junctions with the columns. Transmission due to transverse bending waves in the columns or walls has a lower propagation impedance than for axial waves and is more effectively reflected at the junctions with the floors. Bending waves are sufficiently reflected by the impedance of the floors relative to bending wave impedances in the columns or walls so that the vibratory energy in bending is largely confined to the lower elevations. Axial waves are considered to be the dominant mode for vibration transmission in columns or walls to the upper floors (Conroy, 2008).

2.1. Impedance model of finite height load-bearing column or wall segments

The model assumes that the axial motion within the column or wall structures is relatively uniform over their cross-section and at the junctions with the floors and results in the transmission of bending waves into the floors. Nonuniform axial motion across the cross-section of the rectangular wall segments in the thinner thickness direction corresponds to bending deformation which is more highly attenuated as described above. Nonuniform motion in the larger width direction corresponds to a combination of higher impedance bending and/or shear wave transmission within the plane of the wall, which has been addressed by Craik for extended load-bearing walls (Craik, 1982). Such motions are more complicated in a finite width wall and may possibly be addressed by the transmission of bending in a thick Timoshenko beam, including effects related to transverse shear and rotary inertia. The model here accounts only for the transmission of the higher impedance axial waves characterized by uniform motion over the contact area between the walls and the floors. The wall segments were modeled as equivalent columns involving axial wave transmission based on their cross-sectional area.

The impedance model of axial wave transmission in load-bearing column/wall segments, as illustrated in Figure 3, relates the complex amplitudes of the contact forces, \( f_{c(a) or b,bp} \) and velocities, \( v_{c(a) or b,bp} \), at the top, \( b \), or bottom, \( a \), of the segment which are, in general, complex functions of frequency assuming \( e^{i\omega t} \) time dependence where \( j = \sqrt{-1} \), and \( \omega \) is the circular frequency.

The \( 2 \times 2 \) impedance matrix equations relating the force and velocity amplitudes for the axial wave transmission is given in equation (1)

\[
\begin{bmatrix}
  f_{c(b)} \\
  f_{c(a)}
\end{bmatrix} =
\begin{bmatrix}
  z_c
\end{bmatrix} \begin{bmatrix}
  v_{c(b)} \\
  v_{c(a)}
\end{bmatrix}
\]

(1)

where \( z_c \) is the impedance matrix of a column/wall segment, which can be expressed as equation (2)

\[
 z_c = \begin{bmatrix}
 z_{11c} & z_{12c} \\
 z_{21c} & z_{22c}
\end{bmatrix}
 = \frac{E A k_c}{j \omega \sin(k_c L)} \begin{bmatrix}
 \cos(k_c L) & -1 \\
 -1 & \cos(k_c L)
\end{bmatrix}
\]

(2)

and \( z_{11c}, z_{12c}, z_{21c}, \) and \( z_{22c} \) are impedance expressions in the \( 2 \times 2 \) impedance matrix of column. The index 11, 12, 21, and 22 are locations in the \( 2 \times 2 \) matrix and index \( c \) represents column/wall segment. \( k_c = (\omega/c_v) \) is the axial wavenumber, and \( c_v \) is the complex wave speed for axial waves in the column/wall, defined by \( c_v = \sqrt{E/\rho} \). \( A \) is the
cross-sectional area, and \( L \) is the height of the load-bearing segment. \( \rho \) is the density, and \( E \) is the complex elastic modulus of the column/wall material which accounts for mechanical damping within the column/wall as shown in equation (3)

\[
E = E(1 + j\eta)
\]  

Here, \( \eta \) is the damping loss factor.

The cos and sin factors in equation (2) are due to axial waves with different amplitudes that propagate up and down within the segments and allow for resonant behavior within the column/wall as a whole when combined with the other segments to obtain the system equations.

2.2. Floor impedance models

The point response of a finite floor has been shown by Smith and Lyon (1965) to approach the infinite floor result when the floor is large with many resonances and has damping levels typical of slabs in buildings. The floors act as energy sinks that remove energy transmitting in axial waves within the load-bearing columns/walls. Internal damping within the relatively short column/wall sections between floors is of little consequence because the floors extract significant energy from the transmission.

The input impedance of the floor at the junction with the column/wall segments is complicated. It is assumed that the vertical motion across the cross-section of the column/wall is relatively uniform and characterized by a single junction velocity. The input impedance of the floor for a given uniform motion over an area that matches the cross-section of the column/wall can be quite complicated depending on the extent of the cross-section relative to the wavelength for bending waves in the floor as a function of frequency.

If the contact surface of floor with column/wall junction is small relative to the floor bending wavelength, then it is reasonable to assume that the point impedance expression for the floor applies. At higher frequencies, if the floor bending wavelength becomes comparable with or smaller than the extent of the contact, then the point impedance assumption for floor would not be a good approximation.

An alternate and simplified impedance model of the floor is referred to as the floor beam model, shown in Figure 4(a). The flooring on either side of a wall/column is modeled as beams that extend to infinity away from the wall/column. The assumption is that at frequencies where the floor bending wavelength is not large compared with the maximum length of the contact area between the floor and wall/column, the contact force between the floor and the column/wall can be represented by the transverse force impedance of beams on the either side of the contact. The force resultants are assumed not to vary over the length of the line connection between the floor and wall/column segments, consistent with the modeling of the walls/columns. The width of the beams would equal the maximum length of the contact area.
By symmetry, with identical beams on the either side of the contact, vertical motion at the junction will not produce any rotation (Craik, 1982; Zou et al., 2018). The transverse force impedance of the floor becomes

\[
z_f = \frac{-2Eh_b^3}{\omega} (1 + j)
\]  

(4)

where \( k_f = (\omega/c_f) \) is the bending wavenumber of the floor beam model and \( c_f = \sqrt{EI}/\rho A^{1/2} \) is the bending wave speed. \( \rho \) is the density of floor material, \( A \) is the cross-sectional area, \( I \) is the cross-sectional moment of inertia, and \( E \) is the complex modulus of elasticity given in equation (3).

For the floor point model, where the bending wavelength is large relative to the extent of the contact area, as shown in Figure 4(b), the floor impedance equals the point input impedance of an infinite plate. The infinite floor impedance is constant in frequency and purely real, as shown in equation (5). It represents energy that is extracted from the wave transmission in the column/walls due to radiation of bending waves into building floors

\[
z_f = 8\sqrt{Dp}\frac{h_m}{\rho}
\]  

(5)

where \( D = (Eh_b^3/12(1 - v^2)) \) is the bending stiffness, \( h_m \) and \( h_b \) are floor thicknesses considering an equivalent homogenous flat plate with the same plate mass and bending stiffness of the actual composite slab, that account for composite action in the actual floor system which has girders (Sanayei et al., 2014).

2.3. Assembly of the system of equations for a column/load-bearing wall

Shown in Figure 5, the model describes the transmission in a single column or wall where the total floor vibration is because of contributions of all columns. Sanayei et al. (2012, 2014) measured train-induced vibrations at adjacent columns on a floor and found them to be statistically incoherent so that the vibration transmission in the building can be modeled as the sum of mean square vibration levels contributed by the individual columns. The model predicts the vibration on floor close to the columns where it is assumed that the level at each column is due to the transmission within that column with negligible contributions from the transmission in other columns that propagates across the floor. The model does not predict the total vibration on the open floor because of inputs from all columns.

The input at the foundation is the vertical vibration due to the train pass-by at an individual column/wall. Different columns/walls would experience potentially different input vibration levels (Sanayei et al., 2012). The symbol \( v_k \) represents the train-induced velocity at the junction of floor \( k \) and column/wall segments above and below. Also, shown for the \( k \)th floor are the vertical contact forces, \( f_k^b \) and \( f_{k+1}^a \), between the column segments below and above floor \( k \), respectively.

The impedance model consists of individual column/wall segments and floors whose impedances are given in Sections 2.1 and 2.2. Inputs to the columns/walls in model are in the form of measured vibration levels at either ground or platform elevations. For a building model with \( N \) floors, including the roof, there are \( N \) primary unknown velocities at the junctions between the column/wall segments and the floors apart from the measured input velocity at the column/wall base. The columns/walls and floors at a junction have the same velocity. The internal contact forces between the column/wall segments and the floors are eliminated when forming the system equations only in terms of the unknown junction velocities by summing the contact forces above and below a floor as given in the column/wall impedance expressions into the impedance expression for the floor.

For the 1st floor, index 1, with the input at the base of the first column/wall segment below floor 1, the result is shown in equation (6). For a typical floor, index \( k \), within the
building, the result is given in equation (7), and for the nth floor/roof, index n, with no column segment above equation (8) applies. The index c and f represent the column/wall segment and floor. The subscripts 11, 12, 21, and 22 are the position in the column impedance matrix of equation (2).

For the 1st floor
\[ -z_{12c}^1 v^{\text{input}} = \left( z_{11c}^1 + z_{22c}^1 + z_f^1 \right) v^1 + z_{21c}^1 v^2 \]  
(6)

For the kth floor
\[ 0 = \left( z_{11c}^k + z_{22c}^{k+1} + z_f^k \right) v^k + z_{21c}^{k+1} v^{k+1} + z_{12c}^k v^{k-1} \]
\( (k = 2, \ldots, n-1) \)  
(7)

For the nth floor/roof
\[ 0 = \left( z_{11c}^n + z_f^n \right) v^n + z_{12c}^n v^{n-1} \]  
(8)

The expressions in equations (6)–(8) define the model and can be represented in the matrix form as

\[
\begin{bmatrix}
0 \\
\vdots \\
0 \\
-\frac{1}{z_{12c}^\text{input}}
\end{bmatrix}
= 
\begin{bmatrix}
\left( z_{11c}^1 + z_f^1 \right) & z_{12c}^1 & 0 \\
\vdots & \vdots & \vdots \\
0 & \cdots & \cdots \\
0 & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
\frac{1}{z_{12c}^\text{input}} \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
v^1 \\
\vdots \\
v^n
\end{bmatrix}
\]  
(9)

where k is the story index. The assembled system impedance matrix is of size n by n, that is the number of floors, forming \( F = ZV \). The n junction velocities, V, between the column/wall segments and floors in the building are obtained in terms of the known input at the base of the first column/wall segment by inverting the system impedance matrix in equation (9) as \( V = Z^{-1}F \). As well, an input impedance can be obtained for the column/wall as the ratio of the applied force at its base divided by the input velocity.

The column vertical vibration at the floor junctions radiates bending waves into the open floors where the resulting vibration impacts the buildings’ occupants as feelable vibration. Floor vibration also radiates noise into the buildings’ rooms. The model does not account for the generation of an average floor vibration level in the middle of the floors, including inputs from all columns that contribute to the overall transmission into the building.

The velocities are quickly calculated on a personal computer, for instance, using MATLAB or other modeling applications, with little computational cost or time required to assemble the model. The number of DOF in the model, which equals the number of floors in the building, is significantly less in comparison with, for instance, a finite element model of the building.

3. Applications

Impedance models were developed to characterize vibration transmission in over-track buildings in the Qianhai metro depot in Shenzhen, China. There are two different types of over-track buildings in the depot, which comprises more than 50 buildings. A typical building of each type was selected, including a 4-story low-rise building and a 28-story high-rise building, where their locations within the throat area of the depot are shown in Figure 6.

The throat area of a depot is where the subway trains are parked when not in use, and where maintenance and repair work is carried out. Tracks in the throat area fan out into parking slips and include switches and rail joints. In addition to rolling induced vibration, the track structures are potential sources of vibration generation due to impacts between the wheels and tracks. Train speeds in the throat area are between 10–20 km/h. The test line, at the ground level along the edge of depot, is where trains are evaluated for maintenance and repair purposes and can be operated at speeds up to 80 km/h. As shown in Figure 7, the trains run on the ground level in the Qianhai depot.

3.1. 4-Story over-track low-rise building

3.1.1. Building description. As shown above in Figure 6, a low-rise 4-story office building over the throat area was selected to compare measurement levels with model estimates. Acceleration levels were measured at the track level at locations adjacent to columns located between the tracks that support a thick platform floor that extends throughout the depot. Cars are parked on the platform floor. Columns within the building are supported at the platform level by a heavy transfer structure with beams that span between the ground columns that are not aligned with the building columns.

Acceleration levels were also measured on different floors adjacent to the building columns. The measurements close and adjacent to both the ground and building columns are representative of the axial vibration within the columns at junctions with the floors or ground. Ground-borne vibration from the trains generates axial vibration within the ground columns that transmits into bending vibration within the transfer structure and subsequently into axial vibration.
in the building columns. At junctions with the floors, the axial motion in the building columns radiates bending waves into the floors.

**Figure 8** shows the plan view of the building and identifies the selected interior steel building column that was used for validating the impedance model. The column is encased by cement mortar on its surface. The input to the impedance model is the measured vibration at column base on the platform level. The column extends up from the platform level to the roof with regular geometry. The model does not, at present, account for transmission from the ground level column across the transfer structure and into the building column, as this involves the conversion of axial vibration in the ground column into bending in the transfer structure and subsequently into axial motion in the building column at the platform level.

The concrete masonry unit (CMU) walls in **Figure 8** connect to the concrete casing around the steel column. They are of lower density and elastic modulus and are assumed to have a negligible effect on axial wave transmission within the steel columns. In addition, there is only partial continuity along the top of the CMU wall to the upper floor with no connection of the top steel beam and partial connections to the side column such that vibration transmission between floors through the CMU walls is negligible. As a result, the CMU walls adjacent to the steel column are not accounted for in the impedance model. The column dimensions are much smaller than the wavelength of bending waves in the floor in the frequency range of 4–200 Hz. Therefore, the point impedance expression given in equation (5) is used to model the floors in low-rise building.

**Tables 1 and 2** show the properties of steel columns and concrete floors. The columns are made of square steel tubing with the external dimensions of $a$ and wall thickness $h$. The composite floors consist of the concrete slab over steel beams and steel girders. The equivalent concrete floor
thicknesses $h_k$ and $h_m$ were calculated based on the actual contribution of the concrete slab, steel beams, and girders for stiffness and mass calculations, respectively (Sanayei et al., 2014).

3.1.2. Vibration measurements in the low-rise building. Figure 9 shows measured vertical vibration levels from the platform up and on the roof adjacent to the building column shown in Figure 5. Measured on the floor close to the column, the levels represent the axial vibration within the column at the junctions with the floors. Ambient vibration levels were at least 10 dB below the train induced levels from 6.3 to 63 Hz. The vibration variations from floor to floor are also similar to the measurements in an over-track building as reported by Cao et al. (2018). It is observed that there are two peaks in the vibration levels at about 10 Hz and 31.5 Hz. At present, the origins of these peaks are uncertain. However, the higher frequency peak is consistent with prior measurements of
train-induced vibration in an over-track building in a metro depot in China (Zou et al., 2017).

For the frequency above 31.5 Hz, the buildings considered here, there is an important question concerning the effect of accordion-like resonances due to axial waves within the building columns/walls occurring at these frequencies in this region that are well damped by transmission out from the columns/walls into the flooring. However, in low frequencies, the building bounces together like an integral structure. The dominant frequency is controlled by the soil-structure interaction because the predominant period of the soil is at this lower frequency. Soil resonance occurs during a train pass-by when the period of natural vibration is equal to or close to the predominant period of the soil. In this case, in the region of below 12.5 Hz, the reason of vibration amplification is because of the building resonances at lower frequencies that are close to the frequency of soil resonance. At the higher floor, such as the top floor, part of the vibration energy will gradually spread out along the floor from the intersection between the column and the floor slab. Because there is no vertical load-bearing structure to absorb energy above the floor slab, the energy will be transferred to the floor slab, which is also one of the reasons for the amplification of the higher floor slabs.

In addition, the federal transit administration (FTA) criteria (Federal Transit Administration, 2018) are less stringent at very low frequencies below 8 Hz where special vibration isolation has been provided, and the resonant frequency of the isolation system is very low. The peak velocity levels are less than FTA criteria limit of 72 dB for residential humans in the night, which means the vibration is not feelable. However, the peak velocity levels are larger than criterion curve of VC-B with maximum velocity level of 60 dB (Federal Transit Administration, 2018), which can adversely affect the operation of vibration-sensitive equipment, such as high-power optical microscopes.

### 3.1.3. Comparison of measured and estimated vibrations.

Figure 10(b)–(f) compares measured and estimated vibration levels at the instrumented floors. Ambient vibration levels are shown at each floor. Figure 10(a) shows vibration levels on the platform at the building column shown in Figure 5. The platform vibration was selected as the input because the model describes transmission within the building column above the transfer structure at the platform level. As currently configured, the model does not account for transmission up through the ground columns into and across the transfer structure to the building columns. The equivalent floor impedances account for composite action between steel girders and the concrete slabs, which are higher than the column impedances. The estimated vibration levels above the frequency of 12.5 Hz decrease with floors, which are consistent with measured vibration responses in Figure 9. For the vibration prediction in the frequency of 6.3–12.5 Hz, the estimated vibration levels on the 2nd, 3rd, 4th floors, and roof underestimated vibration levels. The reason for this problem may be because of the calculation of the model did not take into account the nonload-bearing components, such as partition wall. The vibration of nonload-bearing structures may cause the building to resonate at some frequencies. In this case, train-induced vibration generally becomes more significant at about 30–40 Hz and above. For this reason, the estimated vibration levels in the building columns by using the impedance model are considered to agree well with the measured levels throughout the building from the plaza to the roof, which is based on the main purpose of this research to develop a simple practical method to roughly estimate the vibration levels within a building before construction during the design phase. The impedance model, which is based on axial wave transmission in the columns, accurately estimated levels within the selected steel column of the low-rise building.

### 3.2. 28-Story over-track high-rise building

#### 3.2.1. Building description.

A typical 28-story over-track high-rise building shown in Figure 11, was selected to estimate vibration transmission where the building support structure consisted of reinforced concrete load-bearing walls that are aligned from the ground through the full height of the building to the roof. The building is directly over the test line as shown in Figure 6.

Impedance models were generated for the straight wall (S wall) and L shaped wall (L wall) segments at the ground level that are shown in the photos in Figure 12. The wall segments were modeled as equivalent columns involving axial wave transmission based on their cross-sectional area, as described by equations (1) and (2). Tables 3–5 show the properties of the S and L wall segments at different elevations within the building. Bending in the composite concrete floors accounting for the stiffness and mass of girders and beams is described by equivalent thicknesses \( h_k \) and \( h_{int} \), respectively (Sanayei et al., 2014). A uniform equivalent thickness for the composite floors is shown as \( h \) in Table 5.

#### 3.2.2. Vibration measurements in the high-rise building.

Vibration measurements were only measured at the ground level and on the 2nd floor adjacent to the S wall but at the L wall throughout the building from the ground to the 22nd floor. Again, vertical vibration levels were measured on the floors close to the S and L walls with the assumption that they represent the axial transmission vertically within the walls.

The models do not account for transmission due to transverse bending waves within the walls perpendicular to the plane of the walls. In-plane motion within the plane of a wall segment is more complicated and potentially involves in-plane shear deformation within the wall. Models have
been developed for buildings with load-bearing walls between floors, including both in-plane dilatational and shear waves that propagate at different directions within the walls (Craik, 1982). Such models have generally been used to describe transmission at higher frequencies than of concern for this situation with train-induced vibration transmission below 100 Hz. At lower frequency, the widths of the wall segments are comparable with or smaller than the wavelengths for the in-plane motions. The model, as presented, represents in-plane or axial wave transmission in the

Figure 10. Measured and estimated vibrations in low-rise building. (a) Platform, (b) plaza/1st floor, (c) 2nd floor, (d) 3rd floor, (e) 4th floor, and (f) roof.
vertical direction only. As this motion is of higher impedance than transverse bending waves, it is assumed that it adequately describes the transmission into the upper floors of the building.

Figure 13(a) and (b) shows measured vibration levels on the platform and 2nd floor at the S wall and from the 2nd floor to the 22nd floor at the L wall. Measured data at the platform for the L wall were corrupted. Ambient levels were

<table>
<thead>
<tr>
<th>Floor</th>
<th>Height ( l ) (m)</th>
<th>Thickness ( t ) (m)</th>
<th>Length ( w ) (m)</th>
<th>Area ( A ) (m(^2))</th>
<th>( E ) (GPa)</th>
<th>( \rho ) (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground–platform</td>
<td>9.0</td>
<td>0.7</td>
<td>3.3</td>
<td>2.31</td>
<td>34.5</td>
<td>2420</td>
</tr>
<tr>
<td>Platform–plaza</td>
<td>7.0</td>
<td>0.5</td>
<td>3.4</td>
<td>1.70</td>
<td>34.5</td>
<td>2420</td>
</tr>
<tr>
<td>Plaza–2nd floor</td>
<td>5.7</td>
<td>0.4</td>
<td>3.45</td>
<td>1.38</td>
<td>34.5</td>
<td>2420</td>
</tr>
<tr>
<td>2nd floor–3rd floor</td>
<td>3.1</td>
<td>0.4</td>
<td>3.8</td>
<td>1.52</td>
<td>32.5</td>
<td>2400</td>
</tr>
<tr>
<td>3rd floor–roof</td>
<td>2.8</td>
<td>0.2</td>
<td>3.8</td>
<td>0.76</td>
<td>30.0</td>
<td>2385</td>
</tr>
</tbody>
</table>

Table 3. Concrete S wall properties in the high-rise building.
3rd floor–roof 2.8 0.2 6.25 1.25 30.0 2385

Table 5. Floor properties in the high-rise building.

<table>
<thead>
<tr>
<th>Floor no.</th>
<th>Height h (m)</th>
<th>E (GPa)</th>
<th>ρ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platform</td>
<td>0.18</td>
<td>31.5</td>
<td>2390</td>
</tr>
<tr>
<td>Plaza</td>
<td>0.15</td>
<td>31.5</td>
<td>2390</td>
</tr>
<tr>
<td>2nd floor</td>
<td>0.18</td>
<td>34.5</td>
<td>2420</td>
</tr>
<tr>
<td>3rd floor–roof</td>
<td>0.12</td>
<td>30.0</td>
<td>2385</td>
</tr>
</tbody>
</table>

10 dB or more below measured train-induced levels over the full frequency range to 200 Hz. Levels for trains on the test line traveling at 60 km/h were broad over a wide range and peak near 5 Hz and 63 Hz by comparison with the slower trains on the curved tracks in the throat area under the low-rise building. Measured vibration levels on each floor are almost comparable with little reduction from floor to floor in agreement with the measured vibration levels in Figure 9.

3.2.3. Comparison of measured and estimated vibration levels for L and S walls. Figures 14 and 15 show comparisons of measured and estimated vibrations. The platform vibration, Figure 14(a), was used as the input to the base of the impedance model of the S wall. For the L wall estimates, the 2nd floor vibration levels shown in Figure 15(a) were used as the input. Measured levels at the ground level were not used as inputs to the models for transmission in the S and L walls. The ground levels were measured on relatively soft soil close to the walls where it appeared that this resulted in a problem at higher frequencies at and above 80 Hz where the measured motion on the soil surface was decoupled from the wall and did not accurately represent the axial motion within the wall. This issue is discussed in more detail in Section 4.3.

For the S wall, both the estimated vibration levels based on the floor beam model and floor point matched well with the measured vibration levels. For the L wall, the estimated vibration levels on the 5th floor and 8th floor by using floor beam model agreed well with the measured levels, and the floor point model overestimated vibration levels at the frequencies at 40 Hz and above 63 Hz. However, on the 17th and 22nd floors, the differences at lower frequencies are more significant for both floor models, for unknown reasons, and the estimated vibration levels of the floor beam model at higher frequencies match better with measured vibrations than the floor point model. Overall, the floor beam model provided closer estimates of the vibration transmission within the load-bearing walls of the high-rise building.

As discussed in Section 2.2, the selection of an approximate impedance expression for the floors depends on the wavelength of bending waves in the floors relative to width of the wall. Figure 16 shows the bending wavelength of a floor in the high-rise building as a function of frequency by using the floor beam model (Craik, 1982). Wall widths shown in Tables 3 and 4 for the S and L walls, respectively, are comparable with the floor bending wavelength in the important frequency range for train-induced vibration up to 100 Hz. As the wall dimensions are not significantly less than the bending wavelength, the point impedance model of the floor would not be a good approximation in this case. Modeling the floor impedance by beam impedances resulted in a good match to the measured vibrations.

4. General discussion

4.1. Axial wave transmission in the low-rise and high-rise building columns/walls

Figure 17 compares the impedance for axial wave propagation in the building columns of the low-rise building, $z_n = \rho c_n A_n$, with the point impedance for bending deformation in the floors above the platform, equation (4), where both are purely real quantities. The greater value for the column impedance enables the transmission of axial waves up to the building without significant reduction because of the impedance discontinuities at the floor junctions that would reflect axial transmission in the columns. The real valued floor impedance is nonetheless sufficient to extract and dissipate energy from the axial transmission in the columns in the form of bending waves that radiate from junctions out into the floors. The system damping provided by transmission into the floors is primarily responsible for the decreasing levels at higher elevations within the building, as seen in the measured data in Figure 9. Internal mechanical damping within the columns is not sufficient to contribute much to the decrease.
Figure 18 compares the propagation impedance for axial waves in the L wall of the high-rise building with impedance magnitude of the transverse force and point impedance for bending deformation in the floors. The wall impedance is a purely number, which is higher than the floor impedance. The floor impedance is from 3rd floor and all the floors are the same above the 3rd floor. The higher impedance of the wall indicates the vibration transmission in the building with less reduction or a little amplified. The comparison conforms to measured responses in Figure 13.

4.2. Transfer functions

The model estimates vibration levels at the upper floors from the product of the input vibration lower in the building at the ground or platform times a predicted ratio (i.e. transfer function) of vibration at an upper floor divided by the input vibration. Figure 19(a) shows the estimations of building transfer function ratios for the low-rise building for the input in terms of the measured vibration on the platform. To be consistent, the dimensionless value of the ratio is converted by equation (10) to a value in units of dB. As the estimated levels agree well with the measurements, measured transfer
Figure 15. Measured and estimated vibration levels along the L wall. (a) 2nd floor, (b) 5th floor, (c) 8th floor, (d) 17th floor, and (e) 22nd floor.
The curves at different floors are smooth functions of frequency in the range of 200 Hz. There is little evidence of resonance behavior within the full height of the column between the platform and roof. This is because axial waves within the columns excite bending waves that radiate out into the floors and which provide damping that smooths the axial wave transmission within the columns. The floor point input impedance is a real number corresponding to energy dissipation.

As shown in Figure 17, the floor impedance is less than but relatively comparable with the wave propagation impedance or axial waves in the columns. From equation (5), the floor impedance is proportional to the square of its equivalent thickness so that a significant reduction in floor thickness in the prediction would greatly reduce its effect on damping axial wave propagation in the columns. The result, shown in Figure 19(b), reveals the presence of resonant behavior of the floors above the platform. The actual floors provide sufficient damping to effectively smooth resonant behavior in the columns.

Figure 20(a) shows estimates of transfer function to upper floors from the 2nd floor for the L wall in the high-rise building. For the high-rise building with floor impedances included and the beam model for floor impedance, the results exhibit ripples and are not as smooth as observed for the low-rise building. The propagation impedance within the L wall is significantly greater than the floor impedance for the either model, Figure 18, which means that the floors will extract and dissipate proportionately less energy than occurred in the low-rise building. The L wall is a more substantial load-bearing structure in the taller heavier building than the columns in the low-rise building. The presence of resonant behavior is evident at lower frequency with closer spacing as a result of the greater height of the high-rise (Figure 20(b)), 28-story building relative to the shorter, 4-story low-rise building.
5. Conclusions

This study describes the use of impedance models to estimate vibration within over-track buildings induced by train operations in a metro depot in China. Measurements of building vibration during train pass-bys were carried out for two different building configurations for comparison with estimates based on impedance models for each. It was observed that

1. Impedance model estimates of vibration levels in a typical low-rise building with steel-framed columns and a high-rise building with reinforced concrete load-bearing walls compared well with measured levels, with some discrepancies at higher floors in the high-rise building. The model estimates are based on axial wave transmission within the buildings’ columns/walls in response to measured input axial vibration levels at a lower floor.

2. Vibration transfer functions from lower to upper floors were relatively smooth functions of frequency as axial resonances within the columns/walls were well damped because of energy transmission into and dissipation within the building’s floors.

Figure 19. Estimated velocity ratios at upper floors relative to the platform in the low-rise building. (a) With floor impedances included and (b) with floor impedances excluded.

Figure 20. Estimated velocity ratios at upper floors relative to the 2nd floor for the L wall in the high-rise building. (a) With floor impedances included and (b) with floor impedances excluded.
6. Future work

1. An issue is how to provide model inputs in the form of axial vibration levels at the base of the building before construction. There is a need for additional modeling that characterizes the coupling of ground-borne vibration because of trains or other sources into the building foundation system before construction based on either measured levels for existing sources at the building site or past measurements for comparable sources. It also needs to develop fundamental models to solve the predictive accuracy in the very low frequencies.

2. Another effort would be to extend impedance modeling to consider building constructions with significant transfer structures to predict vibration transmission from the ground level upward into building through a transfer floor.

Declaration of conflicting interests

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